أمثلة :

1) $x\,dx + y\,dy + \frac{ydx - x\,dy}{x^2+y^2}$

الحل :

$$\left(x + \frac{y}{x^2+y^2}\right)dx + \left(y - \frac{x}{x^2+y^2}\right)dy = 0$$

$$M(x,y) = x + \frac{y}{x^2+y^2} \quad , \quad N(x,y) = y - \frac{x}{x^2+y^2}$$

$$\Rightarrow \frac{\partial M(x,y)}{\partial y} = 0 + \frac{x^2+y^2-2y^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$\Rightarrow \frac{\partial N(x,y)}{\partial x} = 0 - \frac{(x^2+y^2)-2x^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$

ومنه فالمعادلة التفاضلية تامة.

$$\Rightarrow F(x,y) = \int M(x,y)\,dx + \varphi(y) = \int \left(x + \frac{y}{x^2+y^2}\right)dx + \varphi(y)$$

$$\Rightarrow \boxed{F(x,y) = \frac{x^2}{2} + arc\,tg\left(\frac{x}{y}\right) + \varphi(y)}$$

اشتقاق بالنسبة لـ y

$$\frac{\partial F}{\partial y} = N(x,y) = 0 - \frac{x}{x^2+y^2} + \varphi'(y)$$

$$\Rightarrow y - \frac{x}{x^2+y^2} = -\frac{x}{x^2+y^2} + \varphi'(y) \Rightarrow \varphi'(y) = y \Rightarrow \varphi(y) = \frac{y^2}{2} + c$$

$$\Rightarrow \boxed{F(x,y) = \frac{x^2}{2} + arc\,tg\left(\frac{x}{y}\right) + \frac{y^2}{2} + c}$$

$$[arc\,tg(u)]' = \frac{u'}{1+u^2}$$

2) $(4x-3y-y\sin x)dx+(\cos x-3x-\sin y)dy=0$

الحل :

$$\frac{\partial M(x,y)}{\partial y}=\frac{\partial(4x-3y-y\sin x)}{\partial y}=0-3-\sin x$$

$$\frac{\partial N(x,y)}{\partial x}=\frac{\partial(\cos x-3x-\sin y)}{\partial x}=-\sin x-3$$

ومنه فالمعادلة التفاضلية تامة.

$$\Rightarrow F(x,y)=\int M(x,y)\,dx+\varphi(y)=\int(4x-3y-y\sin x)dx+\varphi(y)$$

$$\boxed{F(x,y)=2x^2-3yx+y\cos x+\varphi(y)}$$

اشتقاق بالنسبة لـ y

$$\frac{\partial F}{\partial y}=N(x,y)=0-3x+\cos x+\varphi'(y)$$
$$\Rightarrow \cos x-3x-\sin x=-3x+\cos x+\varphi'(y)$$
$$\Rightarrow \varphi'(y)=-\sin x\ \Rightarrow \varphi(y)=\cos y+c$$
$$\Rightarrow \boxed{F(x,y)=2x^2-3yx+y\cos x+\cos y+c}$$

3) $(4x^3y^3-2xy)\,dx+(3x^4y^2-x^2)\,dy=0$

الحل :

$$\frac{\partial M(x,y)}{\partial y}=\frac{\partial(4x^3y^3-2xy)}{\partial y}=12x^3y^2-2x$$

$$\frac{\partial N(x,y)}{\partial x}=\frac{\partial(3x^4y^2-x^2)}{\partial x}=12x^3y^2-2x$$

ومنه فالمعادلة التفاضلية تامة.

$$\Rightarrow F(x,y)=\int M(x,y)\,dx+\varphi(y)=\int(4x^3y^3-2xy)\,dx+\varphi(y)$$

$$\boxed{F(x,y)=x^4y^3-x^2y+\varphi(y)}$$

اشتقاق بالنسبة لـ y

$$\Rightarrow \frac{\partial F}{\partial y}=N(x,y)=3x^4y^2-x^2+\varphi'(y)$$
$$\Rightarrow 3x^4y^2-x^2=3x^4y^2-x^2+\varphi'(y)\ \Rightarrow \varphi'(y)=0\Rightarrow \varphi(y)=c$$
$$\Rightarrow \boxed{F(x,y)=x^4y^3-x^2y+c}$$

4) $(x+y+1)\,dx+(x-y^3+3)\,dy=0$

الحل :

$$\frac{\partial M(x,y)}{\partial y}=\frac{\partial(x+y+1)}{\partial y}=1$$

$$\frac{\partial N(x,y)}{\partial x}=\frac{\partial(x-y^3+3)}{\partial x}=1$$

ومنه فالمعادلة التفاضلية تامة.

$$\Rightarrow F(x,y)=\int M(x,y)\,dx+\varphi(y)=\int(x+y+1)\,dx+\varphi(y)$$

$$\boxed{F(x,y)=\frac{x^2}{2}+yx+x+\varphi(y)}$$

اشتقاق بالنسبة لـ y

$$\Rightarrow \frac{\partial F}{\partial y}=N(x,y)=0+x+0+\varphi'(y)$$

$$\Rightarrow x-y^3+3=x+\varphi'(y)\Rightarrow \varphi'(y)=3-y^3\Rightarrow \varphi(y)=3y-\frac{y^4}{4}+c$$

$$\Rightarrow \boxed{F(x,y)=\frac{x^2}{2}+yx+x+3y-\frac{y^4}{4}+c}$$

5) $(2x+3y+4)\,dx+(3x+4y+5)\,dy=0$

الحل :

$$\frac{\partial M(x,y)}{\partial y}=\frac{\partial(2x+3y+4)}{\partial y}=3$$

$$\frac{\partial N(x,y)}{\partial x}=\frac{\partial(3x+4y+5)}{\partial x}=3$$

ومنه فالمعادلة التفاضلية تامة.

$$\Rightarrow F(x,y)=\int M(x,y)\,dx+\varphi(y)=\int(2x+3y+4)\,dx+\varphi(y)$$

$$\boxed{F(x,y)=x^2+3xy+4x+\varphi(y)}$$

اشتقاق بالنسبة لـ y

$$\Rightarrow \frac{\partial F}{\partial y}=N(x,y)=0+3x+0+\varphi'(y)$$

$$\Rightarrow 3x+4y+5=3x+\varphi'(y)\Rightarrow \varphi'(y)=4y+5\Rightarrow \varphi(y)=2y^2+5y+c$$

$$\Rightarrow \boxed{F(x,y)=x^2+3xy+4x+2y^2+5y+c}$$

... انتهت المحاضرة (9) ...