

نشر بعض التوابع البسيطة:

$$1) \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$2) \ln |1-x| = -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

$$3) \frac{1}{(1-x^2)^2} = \sum_{n=1}^{\infty} n \cdot x^{n-1}$$

$$D) \frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n$$

$$2) \frac{1}{1-x^2} = \sum_{n=0}^{\infty} (x^2)^n = \sum_{n=0}^{\infty} x^{2n}$$

$$3) \frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n \cdot x^{2n}$$

$$4) e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\operatorname{sh} x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{2n!}$$

$$\operatorname{ch} x = \sum_{n=0}^{\infty} \frac{x^{2n}}{2n!}$$

$$\operatorname{sh} x = \frac{e^x - e^{-x}}{2} : \operatorname{sh}(0) = 0$$

$$\operatorname{ch} x = \frac{e^x + e^{-x}}{2} : \operatorname{ch}(0) = 1$$

$$\begin{aligned} y &= \operatorname{sh} x \\ y' &= \operatorname{ch} x \\ y'' &= \operatorname{sh} x \\ y''' &= \operatorname{ch} x \\ y^{(4)} &= \operatorname{sh} x \end{aligned}$$

$$\left. \begin{aligned} y(0) &= 0 \\ y'(0) &= 1 \\ y''(0) &= 0 \\ y'''(0) &= 1 \\ y^{(4)}(0) &= 0 \end{aligned} \right\}$$

$$y^{(n)}(0) = \begin{cases} 0 \\ 1 \end{cases}$$

n زوج
 n فرد

$$\Rightarrow \operatorname{sh} x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

افتتاحية، لتابع عند الصفر

$$f(x) = \sin^2 x$$

$$= \frac{1}{2} (1 - \cos 2x)$$

$$= \frac{1}{2} \left(1 - \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{2n!} \right)$$

$$= \frac{1}{2} \left[1 - 1 - \sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n} x^{2n}}{2n!} \right]$$

$$= \frac{1}{2} \left[- \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2^{2n} x^{2n}}{2n!} \right]$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{2n+1} x^{2n}}{2n!}$$

$$- f(x) = x^3 \cos x$$

$$f(x) = x^3 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{2n!}$$

$$- f(x) = \sqrt{\left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2} = \sin \frac{x}{2} + \cos \frac{x}{2}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x}{2}\right)^{2n+1}}{(2n+1)!} + \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x}{2}\right)^{2n}}{(2n)!}$$

$$= 1 + \frac{\frac{x}{2}}{1!} - \frac{\frac{x^3}{2^3}}{2!} + \frac{\frac{x^5}{2^5}}{4!} - \frac{\frac{x^7}{2^7}}{6!} + \dots$$

$$= 1 + \frac{1}{2!} \frac{x^2}{2^2 \cdot 2!} - \frac{x^3}{2^3 \cdot 3!} + \frac{x^4}{2^4 \cdot 4!} + \frac{x^5}{2^5 \cdot 5!} - \dots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^n \cdot n!}$$

1. نهاية، لتتابع عند الصفر