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احسب التكاملات

$$I_1 = \int_0^1 x \, d\left(\ln(x^2+1)\right) \longrightarrow 2 - \frac{\pi}{2}$$

$$I_2 = \int_0^{\pi} \sin 2x \, d(x^2) \longrightarrow -\pi$$

$$I_3 = \int_{-1}^{5} \operatorname{arctg} x \, d(6x) \longrightarrow 30 \operatorname{arctg} 5 - \frac{3\pi}{2} - 3 \ln$$

$$I_4 = \int_{-8}^{2} \frac{1}{2} \, d[\operatorname{ch} 2x] \longrightarrow \frac{1}{2}(\operatorname{ch}4 - \operatorname{ch}16)$$

احسب التكاملين

$$I = (S)\int_{-5}^{1} f(x)\, d(g(x))$$

$$J = (S)\int_{-5}^{1} g(x)\, d(f(x))$$

حيث أن $f(x) = |x|$

$$g(x) = \begin{cases} x+2 & -5 \le x \le -1 \\ x^2 & -1 < x < 0 \\ 3 & x = 0 \\ \ln(x+2) & 0 < x < 1 \\ 2 & x = 1 \end{cases}$$

بين أن التكامل $\int_{-1}^{5} f(x)\, d\,g(x)$ غير موجود إذا علمت أن

$$f(x) = \begin{cases} 1 & x \neq 0 \\ 3 & x = 0 \end{cases}$$

$$g(x) = \begin{cases} 0 & x \neq 0 \\ -2 & x = 0 \end{cases}$$

تمارين فصل ... ب
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اثبت أن ١١

$$I_1 = (s)\int_0^2 x^2 \, d(\ln(x+1)) = \ln 3$$

$$g(x) = \ln(x+1) \qquad x+1 > 0$$

$$x > -1$$

نلاحظ أن $g(x)$ قابل للاشتقاق على $]-1,+\infty[$

$$I_1 = \int_0^2 x^2 \frac{1}{x+1}\,dx$$

$$I_1 = \int_0^2 \frac{x^2-1+1}{x+1}\,dx = \int_0^2 (x-1)\,dx + \int_0^2 \frac{dx}{x+1}$$

$$I_1 = \left[\frac{x^2}{2} - x\right]_0^2 + \left[\ln|x+1|\right]_0^2$$

$$I_1 = [2 - \cancel{2 - 0}] + \ln 3 - \ln 1 = \ln 3$$

$$I_2 = (s)\int_0^{\frac{\pi}{2}} x \, d(\sin x) = \frac{\pi}{2} - 1$$

نلاحظ أن $g(x) = \sin x$ قابل للاشتقاق على $\mathbb{R}$، بالتالي

$$I_2 = \int_0^{\frac{\pi}{2}} x \cos x \, dx$$

x	$+$	$\cos x$
1	$-$	$\sin x$
0		$-\cos x$

$$= \left[x \sin x\right]_0^{\frac{\pi}{2}} + \left[\cos x\right]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi}{2}(1) - 0\right] + [0 - 1] = \frac{\pi}{2} - 1$$

$$I_3 = (S) \int_{-1} x \, d(\arctan x) = 0$$

نفرض أن $g(x) = \arctan x$ قابل للاشتقاق على $]-1,1[$

$$I_3 = \int_{-1}^{1} x \frac{1}{x^2+1} \, dx$$

نفرض أن $\frac{x}{x^2+1}$ تابع فردي و تكاملي

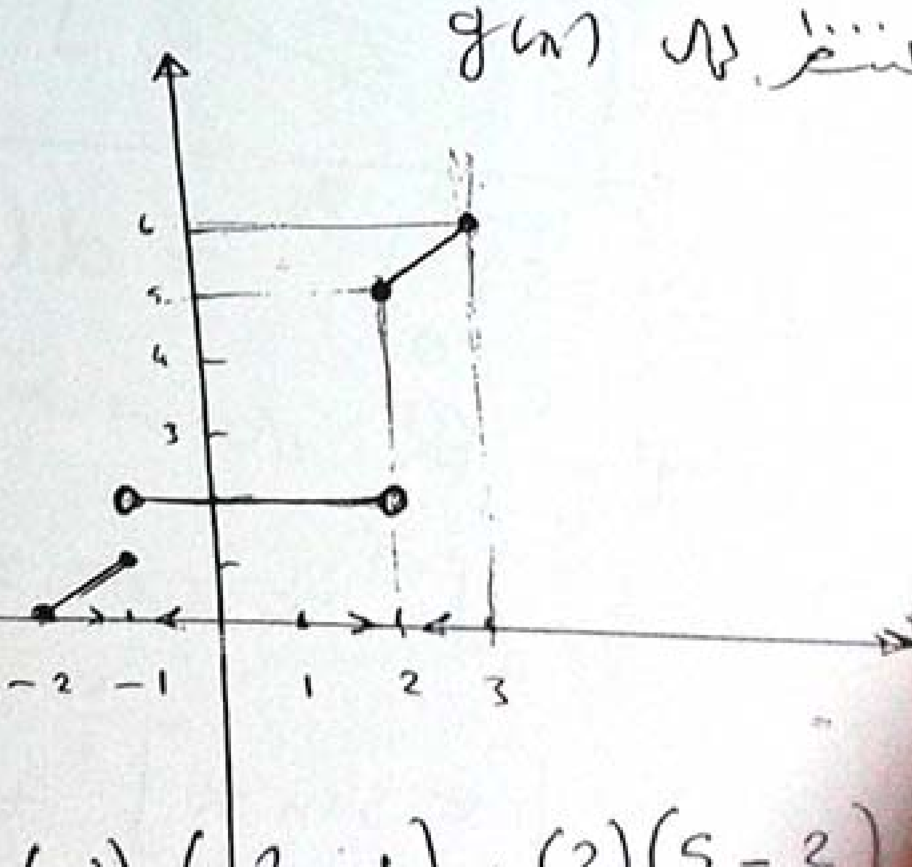
$I_3 = 0$: $[-1,1]$ على

٢/١١١ لنفرض أن

$$g(x) = \begin{cases} x+2 & -2 \le x \le -1 \\ 2 & -1 < x < 2 \\ x+3 & 2 \le x \le 3 \end{cases}$$

$$= (S) \int_{-2}^{3} x \, d(g(x)) = +6$$

فاشث أن :
مستمرة أن $f(x) = x$
لنظر لها $g(x)$



$$= \int_{-2}^{-1} x(1)\,dx + \int_{-1}^{2} x(0)\,dx$$

$$+ \int_{2}^{3} x(1)\,dx +$$

$$f(-1)\left[g(-1+0) - g(-1-0)\right]$$

$$f(2)\left[g(2+0) - g(2-0)\right]$$

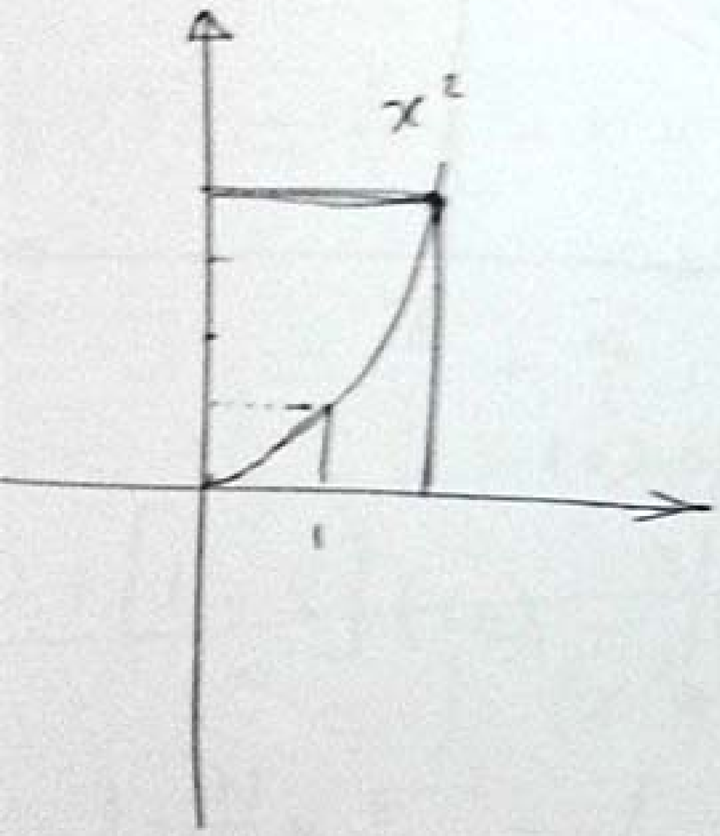
$$= \frac{1}{2}\left[x^2\right]_{-2}^{-1} + 0 + \frac{1}{2}\left[x^2\right]_{2}^{3} + (-1)(2-1) + (2)(5-2)$$

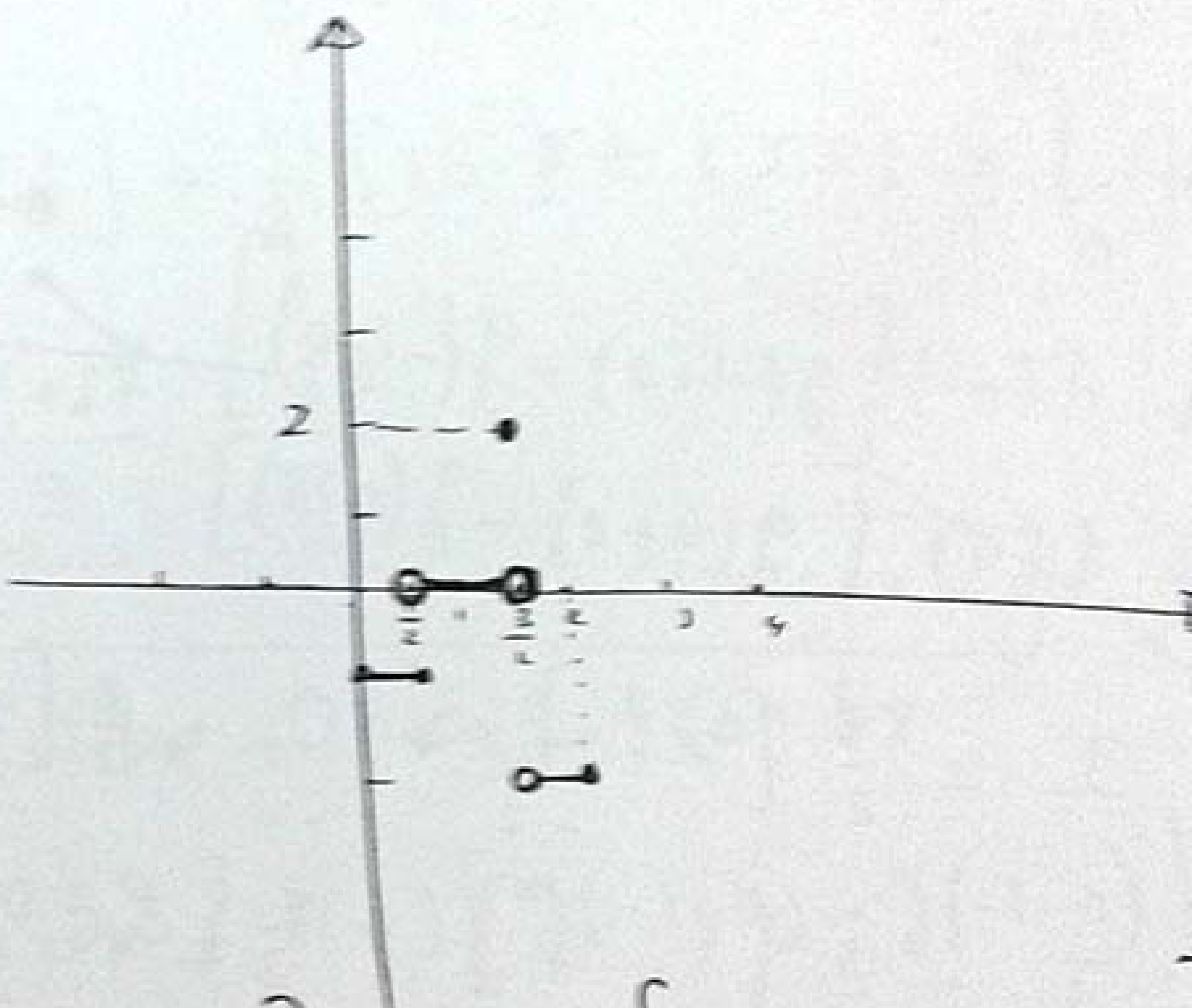
$$= \frac{1}{2}[1-4] + \frac{1}{2}[9-4] - 1 + 6 = -\frac{3}{2} + \frac{5}{2} + 5 = 6$$

$$g(x)=\begin{cases} -1 & 0\le x\le \frac{1}{2} \\ 0 & \frac{1}{2}<x<\frac{3}{2} \\ 2 & x=\frac{3}{2} \\ -2 & \frac{3}{2}<x\le 2 \end{cases}$$ لتكن

$f(x) = x^2$

$$I = (S)\int_0^2 x^2 \, d(g(x)) = \frac{-17}{4}$$ فأثبت أن





$$I = f\left(\tfrac{1}{2}\right)\left[g\left(\tfrac{1}{2}+0\right) - g\left(\tfrac{1}{2}-0\right)\right] + f\left(\tfrac{3}{2}\right)\left[g\left(\tfrac{3}{2}+0\right) - g\left(\tfrac{3}{2}-0\right)\right]$$

$$I = \frac{1}{4}\left[0-(-1)\right] + \frac{9}{4}\left[-2-0\right] = \frac{1}{4} - \frac{18}{4} = \frac{-17}{4}$$

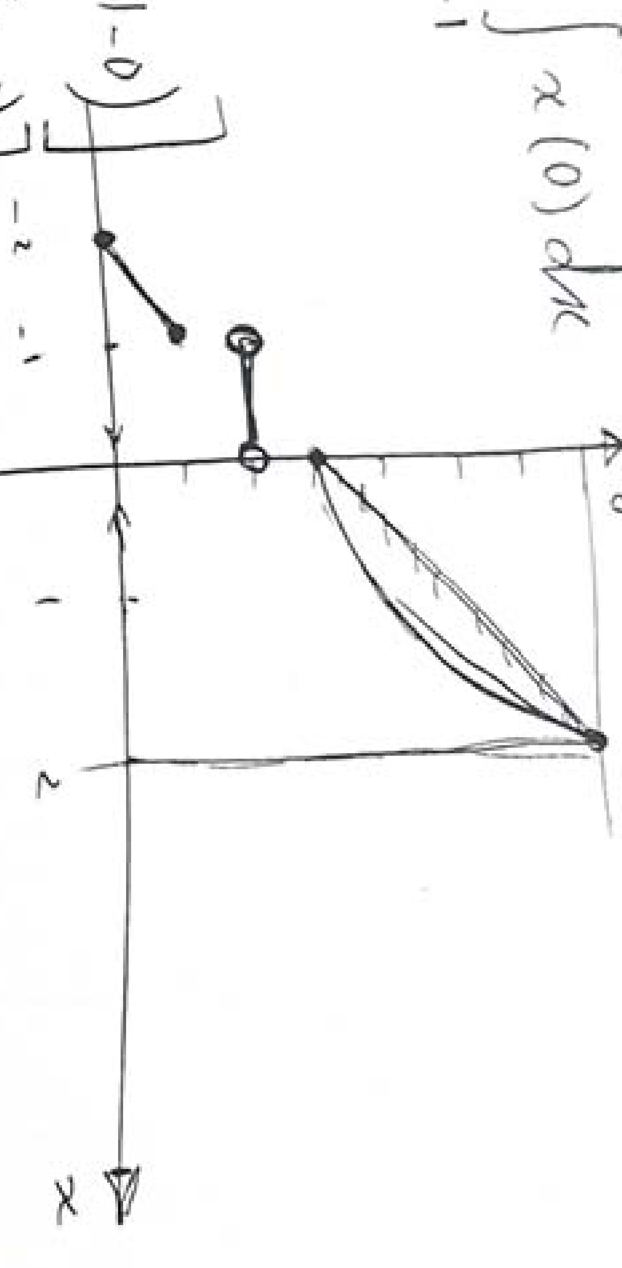
$f(x) = x \qquad g(x) = \begin{cases} x+2 & -2 \le x \le -1 \\ 2 & -1 < x < 0 \\ x^2+3 & 0 \le x \le 2 \end{cases}$

$I = (S)\int_{-2}^{2} x \, d(g(x)) = \frac{17}{6}$

(a) $I = \int_{-2}^{-1} x(1)\,dx + \int_{-1}^{0} x(0)\,dx + \int_{0}^{2} x(2x)\,dx$

$+ f(-1)\left[g(-1+0) - g(-1-0)\right]$

$+ f(0)\left[g(0+0) - g(0-0)\right]$



$I = \frac{1}{2}\left[x^2\right]_{-2}^{-1} + 0 + \frac{2}{3}\left[x^3\right]_{0}^{2} + (-1)\left[2-1\right] + 0$

$I = \frac{1}{2}\left[1-4\right] + \frac{2}{3}\left[8-0\right] - 1 = -\frac{3}{2} + \frac{16}{3} - 1$

$= -\frac{5}{2} + \frac{16}{3} = \frac{-15+32}{6} = \frac{17}{6}.$

(b) $J = (S)\int_{-2}^{2} x^2 \, d(g(x)) = \frac{34}{3}$

(c) $K = (S)\int_{-2}^{2} (x^2+1)\, d(g(x)) = \frac{301}{20}$

4

$$J = \int_{-2}^{-1} x^2(1)\,dx + \int_{-1}^{0} x^2(0)\,dx +$$

$$\int_0^2 x^2(2x)\,dx + f(-1)\left[g(-1+0) - g(-1-0)\right]$$

$$+ f(0)\left[g(0+0) - g(0-0)\right]$$

$$J = \frac{1}{3}\left[x^3\right]_{-2}^{-1} + 0 + \frac{2}{4}\left[x^4\right]_0^2 + 1\left[2-1\right] + 0$$

$$J = \frac{1}{3}\left[-1-(-8)\right] + \frac{1}{2}\left[16\right] + 1$$

$$J = \frac{7}{3} + \frac{9}{1} = \frac{7+27}{3} = \frac{34}{3}$$

$f(x) = x^2+1$

$$k = \int_{-2}^{-1} (x^2+1)(1)\,dx + \int_{-1}^{0} \cancel{(x^2+1)}(0)\,dx$$

$$+ \int_0^2 (x^2+1)(2x)\,dx + f(-1)\left[g(-1+0) - g(-1-0)\right]$$

$$+ f(0)\left[g(0+0) - g(0-0)\right]$$

$$k = \left[\frac{1}{2}x^2 + x\right]_{-2}^{-1} + \left[\frac{1}{2}x^4 + x^2\right]_0^2 + 2\left[2-1\right]$$

$$+ (1)\left[3-2\right]$$

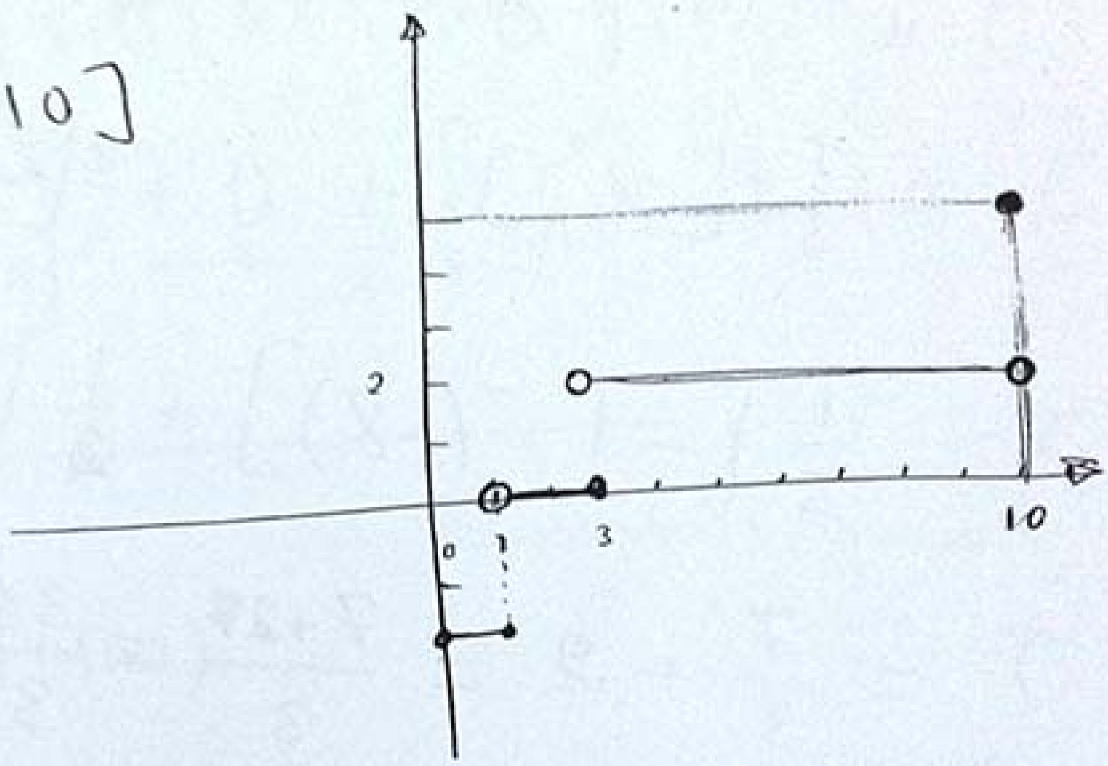
$$k = \left[\left(\frac{1}{2} - 1\right) - \cancel{(2-2)}\right] + \left[(8+4) - (0)\right] + 2 + 1$$

$$k = -\frac{1}{2} + 12 + 3 = 14\frac{1}{2} = \frac{29}{2}.$$

نفرض ان

$$g(x) = \begin{cases} -2 & 0 \le x \le 1 \\ 0 & 1 < x \le 3 \\ 2 & 3 < x < 10 \\ 5 & x = 10 \end{cases}$$

$$f(x) = x^2 \to [0,10]$$



$$I = (S)\int_0^{10} x^2 \, d\,g(x)$$

$$= f(1)\left[g(1+0) - g(1-0)\right] + f(3)\left[g(3+0) - g(3-0)\right]$$

$$+ f(10)\left[g(10) - g(10-0)\right]$$

$$I = 1\left[0-(-2)\right] + 9\left[2-0\right] + 100\left[5-2\right]$$

$$I = 2 + 18 + 300 = 320$$

$$J = \int_0^{10} g(x)\, d(f(x)) = \left[f(x)\,g(x)\right]_0^{10} - \int_0^{10} f(x)\, d(g(x))$$

$$= \left[f(10)\,g(10) - f(0)\,g(0)\right] - 320$$

$$= (100)(5) - 0 - 320 = 500 - 320 = 180$$