

4/5/2014

المحاضرة الثامنة عشر

**جبر الفئات**

لتعرف: لنكن  $(\mathcal{F}_i)_{i \in I}$  أسرة من الفئات، فنزج جبراً وهنئ الفئات المشكل

$$\prod_{i \in I} \mathcal{F}_i : I = \{1, 2\} \rightarrow \mathcal{F}_1 \times \mathcal{F}_2$$

وثنائي، فئته الجبراً وتتألف من:

1- مضاد الاستاد: استناد هذه الفئته جميع الأسر المركبة من المشكل

$$(X_i)_{i \in I} : \forall i \in I ; X_i \in \text{ob}(\mathcal{F}_i)$$

I مجموعة من الأعداد ..

2- صف المورفزمات: ليمر

$$f : (X_i)_{i \in I} \rightarrow (Y_i)_{i \in I}$$

$$f = (f_i)_{i \in I} : \forall i \in I ; f_i \in \mathcal{F}_i(X_i, Y_i)$$

• باعتبار  $I = \{1, 2\}$  التاكيد متروك الفئته ..

**اصرفية** لكي  $\mathcal{F}_1, \mathcal{F}_2$  فئتين .

$$F : \mathcal{F}_1 \rightarrow \mathcal{F}_2$$

دوالها عبارة عن صف يوجب:

$$\text{Hom}_{\mathcal{F}_1}(G) : \mathcal{F}_1 \times \mathcal{F}_1 \rightarrow \text{Sets} \quad \text{دال المصانير} \quad \textcircled{1}$$

$$\text{Hom}_{\mathcal{F}_2}(F) : \mathcal{F}_1 \times \mathcal{F}_2 \rightarrow \text{Sets} \quad \text{دال المصانير} \quad \textcircled{2}$$

**البيانات**

$$\text{Hom}_{\mathcal{F}_1}(G) : \mathcal{F}_1 \times \mathcal{F}_2 \rightarrow \text{Sets} \quad \text{لتعرف} \quad \textcircled{1}$$

$$\forall (A, B) \in \text{ob}(\mathcal{F}_1, \mathcal{F}_2)$$

$$A \in \text{ob}(\mathcal{F}_1) = \text{ob}(\mathcal{F}_1) \quad \text{حيث}$$

$$B \in \text{ob}(\mathcal{F}_2) \Rightarrow G(B) \in \text{ob}(\mathcal{F}_2)$$

$\text{Hom}(G)(A, B) = \mathcal{F}_1(A, G(B))$   
 ∈ Sets  
 من أجل  $\text{Hom}_{\mathcal{F}_1}(G)$

$(A_1, B_1) = (A_2, B_2)$   
 $A_1 = A_2$   
 $B_1 = B_2$   
 $\Rightarrow G(B_1) = G(B_2)$

$(A_1, A_2), (B_1, B_2) \in \text{Cob}(\mathcal{F}_1^\circ, \mathcal{F}_2)$

$(\mathcal{F}_1, \mathcal{F}_2) : (A_1, A_2) \rightarrow (B_1, B_2)$

$\mathcal{F}_1 : A_1 \rightarrow B_1 \in \mathcal{F}_1^\circ(A_1, B_1)$

$\mathcal{F}_2 : A_2 \rightarrow B_2 \in \mathcal{F}_2(A_2, B_2)$

$\text{Hom}_{\mathcal{F}_1}(G)(\mathcal{F}_1, \mathcal{F}_2) : \text{Hom}_{\mathcal{F}_1}(G)(A_1, A_2) \rightarrow \text{Hom}_{\mathcal{F}_2}(G)(B_1, B_2)$

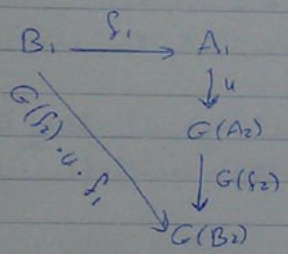
$:\mathcal{F}_1(A_1, G(A_2)) \rightarrow \mathcal{F}_2(B_1, G(B_2))$

تسمى  $u : A \rightarrow B$

تسمى  $F(u) : F(A) \rightarrow F(B)$

$u : A_1 \rightarrow G(A_2) \in \mathcal{F}_1(A_2, G(A_2))$

$\text{Hom}_{\mathcal{F}_1}(G)(\mathcal{F}_1, \mathcal{F}_2)(u) = G(\mathcal{F}_2) \cdot u \cdot \mathcal{F}_1 \in \mathcal{F}_2(B_1, G(B_2))$



$G(\mathcal{F}_2) \rightarrow \mathcal{F}_2$   
 $G(\mathcal{F}_2) : G(A_2) \rightarrow G(B_2)$   
 المورفزمات  $\mathcal{F}_1$  و  $u$  و  $\mathcal{F}_2$   
 من نفس الفئة ولهم  $\mathcal{F}_1$   
 ما التركيب ممكن

نتيجة أنه طبيعي

$(A_1, A_2) \in \mathcal{F}_1^\circ \times \mathcal{F}_2$

$I_{(A_1, A_2)} : (A_1, A_2) \rightarrow (A_1, A_2)$

«لحسابها استخدم الجوار»

$I_{(A_1, A_2)} = (I_{A_1}, I_{A_2}) : \mathcal{F}_1^\circ \times \mathcal{F}_2$

$$\text{Hom}_{\mathcal{F}_1}(G)(I_{(A_1, A_2)}) : \text{Hom}_{\mathcal{F}_1}(G)(A_1, A_2) \rightarrow \text{Hom}(G)(A_1, A_2)$$

$$: \mathcal{F}_1(A_1, G(A_2)) \rightarrow \mathcal{F}_1(A_1, G(A_2))$$

$$w : A_1 \rightarrow G(A_2) \in \mathcal{F}_1(A_1, G(A_2)) \quad \text{... كذا في}$$

$$\text{Hom}_{\mathcal{F}_1}(G)(I_{(A_1, A_2)})(w) = G(I_{A_2}) \cdot w \cdot I_{A_1} \quad \text{... كما}$$

$$= I_{G(A_2)} \cdot w : w$$

$$\text{Hom}_{\mathcal{F}_1}(G)(I_{(A_1, A_2)}) = I_{\text{Hom}_{\mathcal{F}_1}(G)(A_1, A_2)} \quad \text{... وبنفس الطريقة}$$

$$f : A \rightarrow B \quad g : B \rightarrow D \quad \text{... كذا في}$$

... صورتيان القوية  $\mathcal{F}_1 \times \mathcal{F}_2$  حيث

$$\mathcal{F}_1 \times \mathcal{F}_2 \text{ صورة القوية} \quad g \cdot f : A \rightarrow D$$

$$g \cdot f = (g_1, g_2) \cdot (h_1, h_2) = (g_1 \cdot h_1, g_2 \cdot h_2)$$

$$g_1 \cdot h_1 : A_1 \rightarrow D_1 \in \mathcal{F}_1^o(A_1, D_1)$$

$$g_2 \cdot h_2 : A_2 \rightarrow D_2 \in \mathcal{F}_2(A_2, D_2)$$

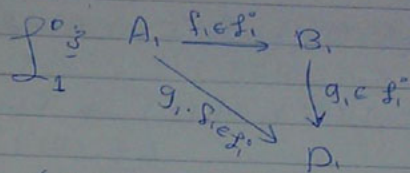
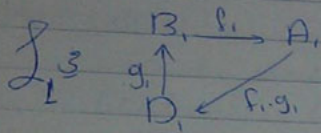
$$\text{Hom}_{\mathcal{F}_1}(G)(g \cdot f) : \text{Hom}_{\mathcal{F}_1}(G)(A_1, A_2) \rightarrow \text{Hom}_{\mathcal{F}_1}(G)(D_1, D_2)$$

$$: \mathcal{F}_1(A_1, G(A_2)) \rightarrow \mathcal{F}_1(D_1, G(D_2))$$

$$(\sigma : (A_1, \dots, G(A_2))) \rightarrow \dots \quad \text{... كذا في}$$

$\sigma: A_1 \rightarrow G(A_2) \in \mathcal{F}_1(A_1, G(A_2))$

$\text{Hom}_{\mathcal{F}_1}(g, f)(\sigma) = G(g_2, f_2) \cdot \sigma \cdot (f_1, g_1)$



$\text{Hom}_{\mathcal{F}_1}(G)(g, f)(\sigma) = G(g_2) \cdot (G(h_2) \cdot \sigma \cdot f_1) \cdot g_1$

$G(h_2) \cdot \sigma \cdot f_1: B \xrightarrow{f_1} A_1 \xrightarrow{\sigma} G(A_2) \xrightarrow{G(h_2)} G(B_2)$   
 $G(h_2) \cdot \sigma \cdot f_1: B \rightarrow G(B_2)$

$\text{Hom}_{\mathcal{F}_1}(G)(g, g_2): \text{Hom}_{\mathcal{F}_1}(G)(B_1, B_2) \rightarrow \text{Hom}_{\mathcal{F}_1}(G)(A_1, D_2)$   
 $= \mathcal{F}_1(B_1, G(B_2)) \rightarrow \mathcal{F}_1(A_1, G(D_2))$

$G(h_2) \cdot \sigma \cdot f_1 \in \mathcal{F}_1(B_1, G(B_2))$

$\text{Hom}_{\mathcal{F}_1}(G)(g)(G(h_2) \cdot \sigma \cdot f_1) = G(g_2) \cdot (G(h_2) \cdot \sigma \cdot f_1) \cdot g_1$

$\text{Hom}_{\mathcal{F}_1}(G)(g, f)(\sigma) = \text{Hom}_{\mathcal{F}_1}(G)(g)(G(h_2) \cdot \sigma \cdot f_1)$   
 $= \text{Hom}_{\mathcal{F}_1}(G)(g)(\text{Hom}_{\mathcal{F}_1}(G)(f)(\sigma))$

$\text{Hom}_{\mathcal{F}_1}(G)(g, f)(\sigma) = \left( \text{Hom}_{\mathcal{F}_1}(G)(g) \cdot \text{Hom}_{\mathcal{F}_1}(G)(f) \right)(\sigma)$

$\forall \sigma$

$\text{Hom}_{\mathcal{F}_1}(G)(g, f) = \text{Hom}_{\mathcal{F}_1}(G)(g) \cdot \text{Hom}_{\mathcal{F}_1}(G)(f)$

X

$$\text{Hom}_{\mathcal{F}_2}(F) : \mathcal{F}_1 \times \mathcal{F}_2 \longrightarrow \text{Sets} \quad (2)$$

$$f_1 = \text{obj } \mathcal{F}_1 \longrightarrow \mathcal{F}_2$$

$$(A, B) \in \text{ob}(\mathcal{F}_1 \times \mathcal{F}_2) \quad \text{نقطة}$$

$$\text{Hom}_{\mathcal{F}_2}(F)(A, B) = \mathcal{F}_2(F(A), B) \in \text{Sets} \quad \text{نقطة}$$

و هو مجموعة التماثل

$$A \in \text{ob}(\mathcal{F}_1) = \text{ob}(F) \longrightarrow F(A) \in \text{ob}(\mathcal{F}_2), B \in \text{ob}(\mathcal{F}_2)$$

$$f : A \longrightarrow B \in \text{Mor}(\mathcal{F}_1 \times \mathcal{F}_2) \quad \text{نقطة}$$

$$A = (A_1, A_2), B = (B_1, B_2), f = (f_1, f_2)$$

$$f_1 : A_1 \longrightarrow B_1 \in \mathcal{F}_1(A_1, B_1)$$

$$f_2 : B_1 \longrightarrow A_1 \in \mathcal{F}_1(B_1, A_1)$$

$$f_2 : A_2 \longrightarrow B_2 \in \mathcal{F}_2(A_2, B_2)$$

$$\text{Hom}_{\mathcal{F}_2}(F)(f) : \text{Hom}_{\mathcal{F}_2}(F)(A) \longrightarrow \text{Hom}_{\mathcal{F}_2}(F)(B)$$

$$: \mathcal{F}_2(F(A), A_2) \longrightarrow \mathcal{F}_2(F(B), B_2)$$

$$\gamma : F(A_1) \longrightarrow A_2 \in \mathcal{F}_2(F(A), A_2) \quad \text{نقطة}$$

$$\text{Hom}(F)(f)(\gamma) = \mathcal{F}_2(\gamma, F(f_1)) \quad \text{نقطة}$$

$$F(B) \xrightarrow{F(f_1)} F(A)$$

$$\begin{array}{ccc}
 & & \downarrow \gamma \\
 & & A_2 \\
 & & \downarrow f_2 \\
 & & B_2 \\
 \nearrow \mathcal{F}_2(\gamma, F(f_1)) & & \\
 & & 
 \end{array}$$

$$I_A: A \rightarrow A \text{ id}$$

$$A = (A_1, A_2) \Rightarrow I_A = (I_{A_1}, I_{A_2})$$

$$\text{Hom}_{\mathcal{F}_2}(F)(I_A) : \text{Hom}_{\mathcal{F}_2}(F)(A_1, A_2) \rightarrow \text{Hom}_{\mathcal{F}_2}(F)(A_1, A_2)$$

$$: \mathcal{F}_2(F(A_1), A_2) \rightarrow \mathcal{F}_2(F(A_1), A_2)$$

$$\forall w: F(A_1) \rightarrow A_2 \in \mathcal{F}_2(F(A_1), A_2)$$

$$\text{Hom}_{\mathcal{F}_2}(F)(I_A)(w) = I_{A_2} \cdot w \cdot F(I_{A_1})$$

$$= w \cdot I_{F(A_1)}$$

$$= w$$

$$\text{Hom}_{\mathcal{F}_2}(F)(I_A) = \underset{\mathcal{F}_2}{I}_{\text{Hom}(F)(A)} \text{ id}$$

$\mathcal{F}_1 \times \mathcal{F}_2$  zell-weise  $f: A \rightarrow B, g: B \rightarrow D$  id  $\times$

$$g \circ f: A \rightarrow D \in \text{Mor}(\mathcal{F}_1 \times \mathcal{F}_2)$$

$$A = (A_1, A_2) \quad , \quad B = (B_1, B_2) \quad , \quad D = (D_1, D_2)$$

$$f = (f_1, f_2) \quad , \quad g = (g_1, g_2)$$

$$g \circ f = \left( \underset{\in \mathcal{F}_1}{g_1 \circ f_1}, \underset{\in \mathcal{F}_2}{g_2 \circ f_2} \right)$$

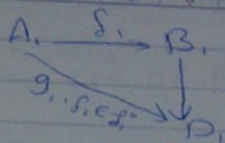
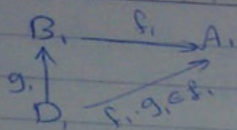
$$\text{Hom}_{\mathcal{F}_2}(F)(g \circ f) : \text{Hom}_{\mathcal{F}_2}(F)(A) \longrightarrow \text{Hom}_{\mathcal{F}_2}(F)(D)$$

$$: \mathcal{F}_2(F(A), A_1) \longrightarrow \mathcal{F}_2(F(D), D_2)$$

$$\forall \lambda: F(A_1) \rightarrow A_2 \in \mathcal{F}_2(F(A_1), A_2)$$

$$\text{Hom}_{\mathcal{F}_2}(F)(g \circ f)(\lambda) = \underbrace{(g_2 \circ f_2)}_{\in \mathcal{F}_2(F(D), D_2)} \cdot \lambda \cdot \underbrace{F(f_1, g_1)}_{f_1, g_1: D_1 \rightarrow A_1 \in \mathcal{F}_1(A_1, D_1)}$$

$$= g_2 \cdot (f_2 \cdot \lambda \cdot F(f_1)) \cdot F(g_1)$$



$$\begin{aligned}
 f_{2, \lambda} \circ F(f_1) &: F(B_1) \xrightarrow{F(f_1)} F(A_1) \xrightarrow{\lambda} A_2 \xrightarrow{f_2} B_2 \\
 f_{2, \lambda} \circ F(h_1) &: F(B_1) \rightarrow B_2
 \end{aligned}$$

$$\begin{aligned}
 \text{Hom}_{f_1}(F)(g) &: \text{Hom}_{f_1}(F)(B) \rightarrow \text{Hom}_{f_2}(F)(D) \\
 &: f_2 \circ F(B_1) \rightarrow B_2 \rightarrow f_2 \circ F(D_1) \rightarrow D_2
 \end{aligned}$$

$$\begin{aligned}
 \text{Hom}_{f_2}(F)(g \circ f)(\lambda) &= \text{Hom}_{f_2}(F)(g)(f_{2, \lambda} \circ F(h_1)) \text{ (as is)} \\
 &= \text{Hom}_{f_2}(F)(g)(\text{Hom}_{f_2}(F)(f)(\lambda)) \\
 &= \text{Hom}_{f_2}(F)(g) \cdot \text{Hom}_{f_2}(F)(f)(\lambda)
 \end{aligned}$$

$$\text{Hom}_{f_2}(F)(g \circ f) = \text{Hom}_{f_2}(F)(g) \cdot \text{Hom}_{f_2}(F)(f)$$

المتجانس