أوجد الحل العام للمعادلة التفاضلية التالية:

(1) $(2y+3xy^3)\,dx + (x+3x^2y^2)\,dy = 0$

$M(x,y) = 2y+3xy^3 \quad ; \quad \frac{\partial M}{\partial y} = 2+9xy^2$

$N(x,y) = x+3x^2y^2 \quad ; \quad \frac{\partial N}{\partial x} = 1+6xy^2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

إذاً المعادلة ليست تامة

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2+9xy^2 - 1 - 6xy^2$

$= 1+3xy^2 \rightarrow$ يوجد تشابه مع N بشكل أكبر (تناسب مع N)

بينما تشابه M لا يوجد إطلاقاً تماماً

$\psi(\ ) = \dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \dfrac{1+3xy^2}{x(1+3xy^2)} = \dfrac{1}{x}$

$\frac{\mu'}{\mu} = \frac{1}{x} \Rightarrow \ln \mu = \ln x \Rightarrow \boxed{\mu = x}$

نضرب المعادلة بـ $(\mu = x)$:

ومنه

$(2xy+3x^2y^3)\,dx + (x^2+3x^3y^2)\,dy = 0$

$M = 2xy+3x^2y^3 \quad , \quad \frac{\partial M}{\partial y} = 2x+9x^2y^2$

$N = x^2+3x^3y^2 \quad , \quad \frac{\partial N}{\partial x} = 2x+9x^2y^2$

إذاً المعادلة تامة

$f(x,y) = \int N(x,y)\,dy + \varphi(x)$

$F(x,y) = \int (x^2+3x^3y^2)\,dy + \varphi(x)$

$F(x,y) = x^2y + \frac{3}{3}x^3y^3 + \varphi(x)$

$\frac{\partial F}{\partial x} = \cancel{2xy} + 3x^2y^3 + \varphi'(x) = M(x,y) = \cancel{2xy} + 3x^2y^3$

$\varphi'(x) = 0 \Rightarrow \varphi(x) = C$

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نعوض في F

$$\boxed{F(x,y) = x^2y + x^3y^3 + c}$$

أدرس عامل التكامل للمعادلة التفاضلية التالية ثم كاملها:

② $(x^2+y)\,dx - x\,dy = 0$

$M = x^2+y \quad , \quad \dfrac{\partial M}{\partial y} = 1$

$N = -x \quad , \quad \dfrac{\partial N}{\partial x} = -1$

المعادلة ليست تامة

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2$$

$$\Rightarrow \psi(x) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-2}{x}$$

$$\frac{\mu'}{\mu} = \frac{-2}{x} \qquad = -2\ln x = \ln x^{-2} = \ln \frac{1}{x^2}$$

$$\ln \mu = \ln \frac{1}{x^2} \Rightarrow \mu = \frac{1}{x^2}$$

نضرب المعادلة بـ $\dfrac{1}{x^2}$

$$\left(1 + \frac{y}{x^2}\right)dx - \frac{1}{x}\,dy = 0$$

$M = 1 + \dfrac{y}{x^2} \quad , \quad \dfrac{\partial M}{\partial y} = \dfrac{1}{x^2}$

$N = -\dfrac{1}{x} \quad , \quad \dfrac{\partial N}{\partial x} = \dfrac{1}{x^2}$

إذا المعادلة تامة

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$$F = \int N(x,y)\,dy + \ell(x)$$

$$= \int -\frac{1}{x}\,dy + \ell(x)$$

$$F(x,y) = -\frac{1}{x}y + \ell(x)$$

$$\frac{\partial F(x,y)}{\partial x} = +\cancel{\frac{1}{x^2}y} + \ell'(x) = M(x,y) = 1 + \cancel{\frac{y}{x^2}}$$

$$\ell'(x) = 1 \Rightarrow \boxed{\ell(x) = x + C}$$

$$\boxed{F(x,y) = -\frac{y}{x} + x + C}$$

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(3) $$\left(2xy + x^2y + \frac{y^3}{3}\right)dx + (x^2 + y^2)\,dy = 0$$

$$M = 2xy + x^2y + \frac{y^3}{3}, \quad \frac{\partial M}{\partial y} = 2x + x^2 + y^2$$

$$N = x^2 + y^2, \quad \frac{\partial N}{\partial x} = 2x$$

اذاً المعادلة ليست تامة

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2x + x^2 + y^2 - 2x = x^2 + y^2$$

$$\psi = \frac{x^2 + y^2}{N} = \frac{x^2 + y^2}{x^2 + y^2} = 1 \Rightarrow$$

$$\frac{\mu'}{\mu} = 1 \Rightarrow \ln \mu = x \Rightarrow \mu = e^x$$

نضرب المعادلة بـ e^x

$$(2xye^x + x^2ye^x + \frac{y^3}{3}e^x)dx + (x^2e^x + y^2e^x)dy = 0$$

$$M = 2xye^x + x^2ye^x + \frac{y^3}{3}e^x \quad , \quad \frac{\partial M}{\partial y} = 2xe^x + x^2e^x + y^2e^x$$

$$N = x^2e^x + y^2e^x \quad , \quad \frac{\partial N}{\partial x} = x^2e^x + 2xe^x + y^2e^x$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

ومنه

إذاً المعادلة تامة

$$F(x,y) = \int N\,dy + \ell(x)$$

$$= \int (x^2e^x + y^2e^x)\,dy + \ell(x) = x^2e^xy + \frac{1}{3}e^xy^3 + \ell(x)$$

$$\frac{\partial f}{\partial x} = 2x\cancel{e^x}y + x^2\cancel{e^x}y + \frac{1}{3}\cancel{e^x}y^3 + \ell'(x) = M = 2\cancel{x}ye^x + \cancel{x^2y}e^x + \frac{\cancel{y^3}}{3}e^x$$

$$\ell'(x) = 0 \Rightarrow \ell(x) = c$$

$$\boxed{F = x^2e^xy + \frac{1}{3}e^xy^3 + c}$$

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(4) $(x + x^4 + x^2y^2)dx + ydy = 0$

$$M = x + x^4 + x^2y^2 \quad , \quad \frac{\partial M}{\partial y} = 2x^2y$$

$$N = y \quad , \quad \frac{\partial N}{\partial x} = 0$$

إذاً ليست تامة

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2x^2y$$

$$\psi(x) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2x^2y}{y} = 2x^2$$

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$$\frac{\mu'}{\mu} = 2x^2$$

$$\ln \mu = \frac{2}{3} x^3$$

$$\mu = e^{\frac{2}{3}x^3}$$

نضرب بعامل التكامل

$$e^{\frac{2}{3}x^3}(x + x^4 + x^2y^2)\,dx + y\,e^{\frac{2}{3}x^3}\,dy = 0$$

$$M = e^{\frac{2}{3}x^2}(x + x^4 + x^2y^2)$$

$$N = y\,e^{\frac{2}{3}x^3}$$

$$F = \int$$

$$F = \frac{1}{2} e^{\frac{2}{3}x^3} y^2 + \ell(x)$$

$$\ell(x) = \int \left[x e^{\frac{2}{3}x^2} + x^2 e^{\frac{2}{3}x^3} \right] dx$$

$$F = \frac{1}{2} e^{\frac{2}{3}x^3} y^2 + \int \left[\qquad \right]$$

بتكملوا الحل...

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5 $$(y'\cos y - x)\,dy + y\,dx = 0$$

$$\mu = \frac{1}{y^2}$$

جبناها

"دوغري"

$$F(x,y) = \frac{x}{y^2} + \sin x + C$$

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(6) $(y+3x^2)\,dx + \left(x + \dfrac{x^4}{y}\right)dy = 0$

"وزاري"

انتهى...

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