$(x \cdot y) = x\,dy + y\,dx$

-5- الحل

أوجد الحل العام للمعادلة التفاضلية التالية:

(1) $(y + 3x^3)\,dx + (x + \frac{x^4}{y})\,dy = 0$

$M = y + 3x^2 \quad , \frac{\partial M}{\partial y} = 1$

$N = x + \frac{x^4}{y} \quad , \frac{\partial N}{\partial x} = 1 + 4\frac{x^3}{y}$

$\left.\right\} \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

فالمعادلة ليست تامة

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = -4\frac{x^3}{y}$

$Ny - Mx = xy + x^4 - (yx + 3x^4) = xy + x^4 - xy - 3x^4$

$= -2x^4$

$\psi(x.y) = \dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{Ny - Mx} = \dfrac{-4\frac{x^3}{y}}{-2x^4} = 2\frac{1}{y.x}$

$\frac{\mu'}{\mu} = \frac{2}{x.y} \Rightarrow \frac{d\mu}{\mu} = \frac{2}{x.y}\,y\,dx + \frac{2}{x.y}\,x\,dy$

$\frac{d\mu}{\mu} = \frac{2\,dx}{x} + 2\frac{dy}{y}$

$\ln|\mu| = 2\ln|x| + 2\ln|y|$

$\ln\mu = \ln x^2 + \ln y^2$

$\ln\mu = \ln x^2.y^2$

$\mu = x^2.y^2$

$\frac{\partial\mu}{\mu} = \frac{2\,d(x.y)}{x.y}$

$\Rightarrow$

$\ln|\mu| = 2\ln|x.y|$

$\ln\mu = \ln(x.y)^2$

$\mu = x^2.y^2$

نضرب المعادلة بعامل التكامل

$(x^2y^3 + 3x^5y^2)\,dx + (x^3y^2 + x^6y)\,dy = 0$

$M = x^2y^3 + 3x^5y^2 \quad , \frac{\partial M}{\partial y} = 3x^2y^2 + 6x^5y$

$N = x^3y^2 + x^6y \quad , \frac{\partial N}{\partial x} = 3x^2y^2 + 6x^5y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

إذاً تامة

$$f(x,y) = \int M\,dx + \ell(y)$$
$$= \int (x^2y^3 + 3x^5y^2)\,dx + \ell(y)$$
$$= \frac{1}{3}x^3y^3 + \frac{3}{6}x^6y^2 + \ell(y)$$

$$\frac{\partial f}{\partial y} = \frac{3}{3}x^3y^2 + \frac{1}{2}2\cdot x^6y + \ell'(y) = N(x,y) = x^3y^2 + x^6y$$

$$\ell'(y) = 0 \Rightarrow \ell(y) = c$$

$$\boxed{F(x,y) = \frac{1}{3}x^3y^2 + \frac{1}{2}x^6y^2 + c}$$

$y'' - 2y' + 10y = e^x$ حل

☆ معادلة برنولي:

أوجد حل المعادلة التفاضلية:

1

$$y' - y = xy^5$$

نفرض أن $y = u.v \Rightarrow y' = u'v + uv'$

نعوض فيه:

$$u'v + uv' - uv = xu^5v^5$$
$$u'v + u(v' - v) = xu^5v^5 \quad \ldots *$$

نعين v بعدم القدار $v' - v$

$$v' - v = 0 \Rightarrow \frac{v'}{v} = 1 \Rightarrow \frac{dv}{v\,dx} = 1 \Rightarrow \frac{dv}{v} = dx \Rightarrow$$

$$\ln v = x \Rightarrow \boxed{v = e^x}$$

تحوص ع *

$$u'e^{x} + u(0) = x u^{5} e^{5x}$$

$$u' = x u^{5} e^{4x}$$

$$\frac{u'}{u^{5}} = x e^{4x}$$

$$\frac{u^{-5} \cdot du}{dx} = x \cdot e^{4} \Rightarrow u^{-5}\, du = x \cdot e^{4} \cdot dx$$

$$\frac{1}{-4} u^{-4} = \int x \cdot e^{4} dx$$

تكامل بالتجزئة $\int x \cdot e^{4} dx$

$$\frac{1}{-4} e^{-4} = \frac{1}{16}(4x-1)e^{4x} + \frac{C}{16}$$

$$u^{4} = \frac{-4}{(4x-1)e^{4x} + C}$$

$$y = u \cdot v$$

$$y^{4} = u^{4} \cdot v^{4}$$

$$y^{4} = \frac{(-4)e^{4x}}{(4x-1)e^{4x} + C}$$

تابع من طرفة y

$$y^{4} = \frac{-4}{(4x-1) + Ce^{-4x}}$$

واجب:

(1) $y' - 2xy = 2x^{3}y^{2}$

(2) $y' - \frac{y}{x} = e^{x}y^{2}$

ع برنولي بطريقة الدكتور ، الطريقة التي أوضحتها بالحل

انتهت.