[1] $y'' - 5y' + 6y = 1$ حل $y(0) = y'(0) = 0$

نأخذ لابلاس: $\mathcal{L}[y''] - 5\mathcal{L}[y'] + 6\mathcal{L}[y] = \mathcal{L}[1]$

$[s^2Y(s) - sy(0) - y'(0)] - 5[sY(s) - y(0)] + 6Y(s) = \frac{1}{s}$

$[s^2 - 5s + 6]Y(s) = \frac{1}{s}$

$Y(s) = \frac{1}{s(s^2-5s+6)} = \frac{1}{s(s-3)(s-2)}$

$= \frac{A}{s} + \frac{B}{s-3} + \frac{C}{s-2}$

نوحد المقامات ونقارن

$A = \frac{1}{6}, B = \frac{1}{3}, C = -\frac{1}{2}$

$\mathcal{L}^{-1}[Y(s)] = A\mathcal{L}^{-1}[\frac{1}{s}] + B\mathcal{L}^{-1}[\frac{1}{s-3}] + C\mathcal{L}^{-1}[\frac{1}{s-2}]$

$y(t) = A[1] + Be^{3t} + Ce^{2t}$

$y(t) = \frac{1}{6} + \frac{1}{3}e^{3t} - \frac{1}{2}e^{2t}$

اعادة التوابيت

نوحد المقامات ونقارن

$A(s-3)(s-2) = As^2 - 5As + 6A$

$As^2 = 0, \ 6A = 1 \Rightarrow A = \frac{1}{6}$

نعوض $s \to 2$ نجد أن

$-2C = 1 \Rightarrow C = -\frac{1}{2}$

ومنه $B = \frac{1}{3}$

[2] $y'' + y' = te^t$ حيث $y(0) = y'(0) = 0$

$\mathcal{L}[y''] + \mathcal{L}[y'] = \mathcal{L}[te^t]$

$[s^2Y(s) - sy(0) - y'(0)] + [sY(s) - y(0)] = \mathcal{L}[te^t]$

حيث $\mathcal{L}[te^t]$ يحول إلى

$f(t) = t \Rightarrow F(s) = \frac{1}{s^2}$

$\mathcal{L}[te^t] = \frac{1}{(s-1)^2}$

$\Rightarrow [s^2 + s]Y(s) = \frac{1}{(s-1)^2}$

$Y(s) = \frac{1}{(s^2+s)(s-1)^2} = \frac{1}{s(s+1)(s-1)^2}$

بتفكيك الكسور إلى

$\frac{1}{s(s+1)(s-1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-1} + \frac{D}{(s-1)^2}$

$A = 1$

$B = -\frac{1}{4}$

$C = -\frac{3}{4}$

$D = \frac{1}{2}$

$$L^{-1}[Y(s)] = AL^{-1}\left[\frac{1}{s}\right] + BL^{-1}\left[\frac{1}{s+1}\right] + CL^{-1}\left[\frac{1}{s-1}\right] + DL^{-1}\left[\frac{1}{(s-1)^2}\right]$$

$$y(t) = A[1] + Be^{-t} + Ce^{t} + Dte^{t}$$

$$y(t) = 1 + \frac{1}{4}e^{-t} - \frac{3}{4}e^{t} + \frac{1}{2}te^{t}$$

$$y(t) = 1 - \frac{1}{4}e^{-t} + \left[\frac{1}{2}t - \frac{3}{4}\right]e^{t}$$

[3] $y'' - 9y = t$ ⟵ $y(0) = 1,\ y'(0) = -3$

$$L[y''] - 9L[y] = L[t]$$

$$[s^2Y(s) - sy(0) - y'(0)] - 9Y(s) = \frac{1}{s^2}$$

$$[s^2Y(s) - s + 3] - 9Y(s) = \frac{1}{s^2}$$

$$[s^2 - 9]Y(s) - s + 3 = \frac{1}{s^2}$$

$$[s^2 - 9]Y(s) = \frac{1}{s^2} + s - 3 \Rightarrow Y(s) = \frac{1}{s^2(s^2-9)} + \frac{s-3}{s^2-9}$$

توحيد المقامات

$$Y(s) = \frac{1}{s^2(s-3)(s+3)} + \frac{1}{s+3}$$

$$\frac{As(s-3)(s+3)}{s^2(s-3)(s+3)} + \frac{B(s-3)(s+3)}{s^2(s-3)(s+3)} + \frac{Cs^2(s+3)}{s^2(s-3)(s+3)} + \frac{Ds^2(s-3)}{s^2(s-3)(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-3} + \frac{D}{s+3} + \frac{1}{s+3}$$

بالمقارنة

$$A = 0,\ B = -\frac{1}{9},\ C = \frac{1}{54},\ D = \frac{-1}{54}$$

بالتعويض $s \to 3$ نجد

$1 = C(3)^2(6)$

$1 = C$

بالتعويض $s \to -3$ نجد

$1 = -27D - 3D(9)$

$D = \frac{-1}{54}$

$s \to 0$

$-3B(3) = 1$

$B = \frac{-1}{9}$

$A = 0$

$$L^{-1}[Y(s)] = AL^{-1}\left[\frac{1}{s}\right] + BL^{-1}\left[\frac{1}{s^2}\right] + CL^{-1}\left[\frac{1}{s-3}\right] + DL^{-1}\left[\frac{1}{s+3}\right] + L^{-1}\left[\frac{1}{s+3}\right]$$

$$y(t) = A[1] + Bt + Ce^{+3t} + De^{-3t} + e^{-3t}$$

$$y(t) = -\frac{1}{9}t + \frac{1}{54}e^{3t} - \frac{1}{54}e^{-3t} + e^{-3t}$$

$$y(t) = -\frac{1}{9}t + \frac{1}{54}e^{3t} + \left[1 - \frac{1}{54}\right]e^{-3t}$$

$$y(t) = -\frac{1}{9}t + \frac{1}{54}e^{3t} + \frac{53}{54}e^{-3t}$$

5) $y'' - 2y' + y = 4$, $y(0) = 4,\ y'(0) = 2$

$$L[y''] - 2L[y'] + L[y] = L[4]$$

$$[s^2Y(s) - sy(0) - y'(0)] - 2[sY(s) - y(0)] + sY(s) = \frac{4}{s}$$

$$[s^2Y(s) - 4s - 2] - 2[sY(s) - 4] + Y(s) = \frac{4}{s}$$

$$s^2Y(s) - 4s - 2 - 2sY(s) + 8 + Y(s) = \frac{4}{s}$$

$$[s^2 - 2s + 1]Y(s) - 4s + 6 = \frac{4}{s}$$

$$[s^2 - 2s + 1]Y(s) = \frac{4}{s} + 4s - 6$$

$$[s^2 - 2s + 1]Y(s) = \frac{4s^2 - 6s + 4}{s} = \frac{4s^2 - 6s + 4}{s(s-1)(s-1)} = \frac{4s^2 - 6s + 4}{s(s-1)^2}$$

$$\frac{4s^2 - 6s + 4}{s(s-1)^2} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

بتوحيد المقامات والمقارنة نجد أن

$A = 4,\ B = 0,\ C = 2$

$$L^{-1}[Y(s)] = L^{-1}\left[\frac{A}{s}\right] + L^{-1}\left[\frac{B}{s-1}\right] + L^{-1}\left[\frac{C}{(s-1)^2}\right]$$

$$y(t) = AL^{-1}\left[\frac{1}{s}\right] + BL^{-1}\left[\frac{1}{s-1}\right] + CL^{-1}\left[\frac{1}{(s-1)^2}\right]$$

$$y(t) = A[1] + Be^t + cte^t$$

$$y(t) = 4 + 2te^t$$

بتوحيد المقامات نجد

$$A(s^2 - 2s + 1)$$
$$+ B(s)(s-1)$$
$$+ C(s) = 4s^2 - 6s + 4$$

$$As^2 - 2As + A + Bs^2 - B + Cs = 4s^2 - 6s + 4$$

بالمقارنة نجد

$$A + B = 4 \quad \text{--- (1)}$$
$$-2A + C = -6 \quad \text{--- (2)}$$
$$A - B = 4 \quad \text{--- (3)}$$

من (3) و (1) نجد أن $B = 0$ و $A = 4$

$$\Rightarrow C - 8 = -6 \Rightarrow C = 2$$

Notes

Appointment

[5] $y'' + 2y' + y = e^{-2t}$; $y(0) = y'(0) = 0$

$$\mathcal{L}[y''] + 2\mathcal{L}[y'] + \mathcal{L}[y] = \mathcal{L}[e^{-2t}]$$

$$s^2Y(s) - sy(0) - y'(0) + 2[sY(s) - y(0)] + 5Y(s) = \frac{1}{s+2}$$

$$s^2Y(s) + 2sY(s) + 5Y(s) = \frac{1}{s+2}$$

$$(s^2 + 2s + 1)Y(s) = \frac{1}{s+2}$$

$$\Rightarrow Y(s) = \frac{1}{(s+2)(s^2+1)^2} = \frac{A}{s+2} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

نعوض عن الثوابت بحيث أن $A = 1, B = -1, C = 1$

$$\mathcal{L}^{-1}[Y(s)] = A\mathcal{L}^{-1}\left[\frac{1}{s+2}\right] + B\mathcal{L}^{-1}\left[\frac{1}{s+1}\right] + C\mathcal{L}^{-1}\left[\frac{1}{(s+1)^2}\right]$$

$$y(t) = Ae^{-2t} + Be^{-t} + Cte^{-t}$$

$$y(t) = e^{-2t} - e^{-t} + te^{-t}$$

$$y(t) = e^{-2t} + (t-1)e^{-t}$$

طريقة ثانية لتعريف الكسور للتمرين السابق

$$\frac{4s^2 - 6s + 4}{s(s-1)^2} = \frac{4(s^2 - 2s + 1) + 2s}{s(s-1)^2}$$

لإيجاد الثوابت نوحد المقامات

$$1 = A(s+1)^2 + B(s+2)(s+1) + C(s+2)$$

$$1 = As^2 + 2As + A + Bs^2 + 3Bs + 2B + Cs + 2C$$

$$As^2 + Bs^2 + 2As + 3Bs + Cs + A + 2B + 2C = 1$$

بالمقارنة نجد:

① $A + B = 0 \Rightarrow \boxed{A = -B}$

② $2A + 3B + C = 0 \Rightarrow 2A - 3A + C = 0 \Rightarrow -A = -C \Rightarrow \boxed{A = C}$

③ $A + 2B + 2C = 1$

نعوض ② في ③

$$A - 2A + 2A = 1 \Rightarrow A = 1 = C \Rightarrow B = -1$$