معادلات تفاضلية c
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المحاضرة السادسة

* الخواص الأساسية لتحويلات لابلاس

① $L[\alpha f(t) + \beta g(t)] = \alpha L[f(t)] + \beta L[g(t)] = \alpha F(s) + \beta G(s)$

② $L[A] = \frac{A}{s}$ , $A \in \mathbb{R}$

③ $L[e^{at}] = \frac{1}{s-a}$ : $a \in \mathbb{R}$ , $s > a$

④ $L[\sin at] = \frac{a}{s^2+a^2}$ : $s > 0$

⑤ $L[\cos aT] = \frac{s}{s^2+a^2}$ :

⑥ $L[\sh aT] = \frac{a}{s^2-a^2}$ , $s > a$ / ⑦ $L[\ch at] = \frac{s}{s^2-a^2}$ , $s > a$

⑧ $L[t^n] = \frac{n!}{s^{n+1}}$ $\left\{ \begin{array}{l} L[t] = \frac{1}{s^2} \\ L[t^2] = \frac{2}{s^3} \end{array} \right.$

⑨ $L[f'(t)] = sF(s) - f(0)$

⑩ $L[f''(t)] = s^2F(s) - sf(0) - f'(0)$

⑪ $L\left[\frac{\partial f}{\partial t}\right] = sF(x,s) - F(x,0)$

⑫ $L\left[\frac{\partial^2 f}{\partial t^2}\right] = s^2F(x,s) - sf(x,0) - f'(x,0)$

⑬ $L\left[\frac{\partial f}{\partial x}\right] = \frac{\partial F(x,s)}{\partial s}$ , ⑭ $L\left[\frac{\partial^2 f}{\partial x^2}\right] = \frac{\partial^2 F(x,s)}{\partial s^2}$

⑮ $L[t^n f(t)] = (-1)^n \frac{\partial^n F(s)}{\partial s^n}$

⑯ $L[e^{at} f(t)] = F(s-a)$ : $s > 0$

Example: $L[3e^{-4t}] = 3L[e^{-4t}] = \frac{3}{s+4}$

$L[2t^2] = 2L[t^2] = 2 \cdot \frac{2!}{s^{2+1}} = \frac{4}{s^3}$

$L[4\cos 5t] = 4L[\cos 5t] = 4\frac{s}{s^2+25}$

$L[\sin \pi t] = \frac{\pi}{s^2+\pi^2}$

$L[e^{3t}\sin 4t] = F(s-a)$

$\hookrightarrow f(t) = \sin 4t \Rightarrow F(s) = \frac{4}{s^2+16}$

$\Rightarrow L[e^{3t}\sin 4t] = \frac{4}{(s-a)^2+16}$

$= \frac{4}{(s-3)^2+16}$

$L[t^2 e^{-2t}] = f(t) = t^2 \Rightarrow F(s) = \frac{2}{s^3} \Rightarrow L[t^2 e^{-2t}] = F(s-a)$

$= \frac{2}{(s-a)^2} = \frac{2}{(s+2)^3}$

$L[t\sin 2t] = (-1)^n \frac{d^n F(s)}{ds^n}$

$\begin{cases} f(t) = \sin 2t \Rightarrow F(s) = \frac{2}{s^2+4} \\ L[t\sin 2t] = (-1)^1 \frac{dF(s)}{ds} \end{cases}$

$= -\left(\frac{2}{s^2+4}\right)' = -2\left(\frac{-2s.2}{(s^2+4)^2}\right) = \frac{4}{(s^2+4)^2}$

$L[t^2 \sin 2t] =$

$f(t) = \sin 2t \Rightarrow F(s) = \frac{2}{s^2+4}$

$L[t^2 \sin 2t] = (-1)^2 \frac{d^2 F(s)}{ds^2} = \left(\frac{2}{s^2+4}\right)'' = \left(\frac{-4}{(s^2+4)^2}\right)' = \dots$ ect