$$\beta - pq = 0 \quad \cdots (*)$$

$$\frac{\partial F}{\partial x} = 0,\ \frac{\partial F}{\partial y} = 0,\ \frac{\partial F}{\partial \beta} = 1,\ \frac{\partial F}{\partial p} = -q,\ \frac{\partial F}{\partial q} = -p$$

$$\frac{\partial x}{\partial p} = \frac{\partial y}{\partial q} = \frac{\partial \beta}{p\frac{\partial F}{\partial p} + q\frac{\partial F}{\partial q}} = \frac{-dp}{\frac{\partial F}{\partial x} + p\frac{\partial F}{\partial \beta}} = \frac{-dq}{\frac{\partial F}{\partial y} + q\frac{\partial F}{\partial \beta}}$$

$$\frac{dx}{-q} = \frac{dy}{-p} = \frac{d\beta}{-pq - qp} = \frac{-dp}{0+p} = \frac{-dq}{0+q}$$

① ② ③ ④ ⑤

نريد تكامل واحد فقط !!

$$4 = 5$$

$$-\frac{dp}{p} = -\frac{dq}{q}$$

$$\frac{dp}{p} = \frac{dq}{q} \Rightarrow \ln(p) = \ln(q) + \ln(a) \Rightarrow p = aq \quad \cdots (*)$$

• نعوض في ① لأننا نريد p و q بدلالة x, y, β

$$\beta - aq^2 = 0 \Rightarrow q^2 = \frac{\beta}{a} \Rightarrow \boxed{q = \sqrt{\frac{\beta}{a}}}$$

$$\boxed{p = \sqrt{a\beta}}$$ ← نعوض q و (*)

• نعوض p, q بالعلاقة التفاضلية الكلية: $d\beta = p\,dx + q\,dy$

$$d\beta = \sqrt{a\beta}\,dx + \sqrt{\frac{\beta}{a}}\,dy$$

$$\frac{d\beta}{\sqrt{\beta}} = \sqrt{a}\,dx + \frac{1}{\sqrt{a}}\,dy$$

$$2\sqrt{\beta} = \sqrt{a}\,x + \frac{y}{\sqrt{a}} + b$$

$$4\beta = \left(\sqrt{a}\,x + \frac{y}{\sqrt{a}} + b\right)^2$$

$$\boxed{\beta = \frac{1}{4}\left(\sqrt{a}\,x + \frac{y}{\sqrt{a}} + b\right)^2}$$ الحل العام

ملاحظة هامة ...

• لو طلب منا الحل الخاص ...

نعد الوصول للحل العام نجعل b تابع لـ a وذلك بكتابة

$$b = t(a)$$

في الحل العام هو:

$$\beta = \frac{1}{4}\left(\sqrt{a}\,x + \frac{y}{\sqrt{a}} + t(a)\right)^2$$

• لو طلب منا الحل الخاص فإننا نعوض لـ b قيمة معينة مثلاً

$$b = 2a$$

• لو طلب منا الحل الشاذ نوعه مختلف ... لا نوع له

$$(\beta + \rho x)^2 - 9 = 0 \quad \cdots ①$$

$$\frac{\partial F}{\partial x} = 2\rho(\beta + \rho x) \quad , \quad \frac{\partial F}{\partial y} = 0$$

$$\frac{\partial F}{\partial \beta} = 2(\beta + \rho x) \quad , \quad \frac{\partial F}{\partial \rho} = 2x(\beta + \rho x)$$

$$\frac{\partial F}{\partial 9} = -1$$

$$\frac{dx}{2x(\beta+\rho x)} = \frac{dy}{-1} = \frac{d\beta}{\rho 2x(\beta+\rho x) - 9} = \frac{-d\rho}{2\rho(\beta+\rho x) + 2\rho(\beta+\rho x)}$$

① ② ③ ④

$$= \frac{-9}{2\rho(\beta+\rho x)}$$

⑤

① = ④

$$\frac{dx}{2x(\beta+\rho x)} = \frac{-d\rho}{4\rho(\beta+\rho x)} \Longrightarrow \frac{-d\rho}{2\rho} = \frac{dx}{x} \Longrightarrow \frac{d\rho}{\rho} = -2\frac{dx}{x}$$

$$\Longrightarrow \ln(\rho) = \ln\left(\frac{1}{x^2}\right) + \ln(a)$$

- $-2\frac{dx}{x} = -2\ln|x| = \ln|x|^{-2} = \ln\frac{1}{x^2}$
- $\ln\left(\frac{1}{x^2}\right) + \ln(a) \cdot \ln\left(\frac{1}{x^2}\cdot a\right) = \ln\left(\frac{a}{x^2}\right)$

$$\Longrightarrow \rho = \frac{a}{x^2} \quad \cdots ⑥$$

نعوض في ①

$$\left(\beta + \frac{a}{x}\right)^2 - 9 = 0$$

$$9 = \left(\beta + \frac{a}{x}\right)^2$$

- نعوض بالمعادلة العامة الكلية

$$d\beta = \rho\, dx + 9\, dy$$

$$d\beta = \frac{a}{x^2}dx + \left(\beta + \frac{a}{x}\right)^2 dy$$

هنا سأضل في كتابة مع الطرف الثاني بحيث لو $d(\beta + \frac{a}{x})$ نفرض ك $d\beta - \frac{a}{x^2}$

$$d\beta + \frac{a}{x^2}dx = (\beta + \frac{a}{x})^2 dy$$

$$\hookrightarrow d(\beta + \frac{a}{x}) = (\beta + \frac{a}{x})^2 dy$$

$$dy = \frac{d(\beta + \frac{a}{x})}{(\beta + \frac{a}{x})^2}$$

نفرض $t = \beta + \frac{a}{x}$

$$dy = \frac{dt}{t^2} \Longrightarrow y + b = \frac{-1}{t}$$

$$y + b = \frac{-1}{\beta + \frac{a}{x}}$$

$$\beta + \frac{a}{x} = \frac{-1}{y+b}$$

$$\Longrightarrow \boxed{\beta + \frac{a}{x} + \frac{1}{y+b} = 0}$$

وهو الحل النهائي ...

وظيفة صامة جداً (ص 256 بالكتاب) + $(\beta + 9y)^2 - P = 0$

وسأرفق الحل بالمحاضرة القادمة إن شاء الله ...

تعريف: أو حد الحل التام) للمعادلة التفاضلية من الشكل

$$Pq - Px - qy = 0 \quad \text{---} \ (1)$$

$$\frac{\partial f}{\partial x} = -P, \ \frac{\partial f}{\partial y} = -q, \ \frac{\partial f}{\partial z} = 0, \ \frac{\partial f}{\partial P} = q - x, \ \frac{\partial f}{\partial q} = P - y$$

$$\frac{dx}{q-x} = \frac{dy}{P-y} = \frac{dz}{P(q-x) + q(P-x)} = \frac{-dP}{-P} = \frac{-dq}{-q}$$

(1) (2) (3) (4) (5)

(4) = (5)

$$\frac{dP}{P} = \frac{dq}{q} \longrightarrow \ln(P) = \ln(q) + \ln(a)$$

$$\boxed{P = aq} \quad \text{---} \ (*)$$

$$aq^2 - aqx - qy = 0$$

$$q(aq - ax - y) = 0$$

إما $q = 0$

$$aq - ax - y = 0 \Longrightarrow \boxed{q = \frac{ax + y}{a}}$$

$$\boxed{P = ax + y}$$

$$dz = P\,dx + q\,dy$$

$$= (ax+y)\,dx + \left(\frac{ax+y}{a}\right) dy$$

$$= ax\,dx + y\,dx + x\,dy + \frac{y}{a}\,dy$$

$$= ax\,dx + d(y.x) + \frac{y}{a}\,dy$$

$$z = \frac{a}{2}x^2 + xy + \frac{y^2}{2a} + b$$