60 د.س تمارين الفصل الثاني س٣ رياضيات - علمي

١٢ تمارين الفصل الثاني

نثبت أن

$$I_1 = (S)\int_0^2 x^2\, d(\ln(x+1)) = \ln 3$$

$g(x) = \ln(x+1)$ $\qquad x+1>0$

نلاحظ أن $g(x)$ قابل للاشتقاق $\qquad x>-1$

$]-1,+\infty[$

$$I_1 = \int_0^2 x^2 \frac{1}{x+1}\,dx$$

$$I_1 = \int_0^2 \frac{x^2-1+1}{x+1}\,dx = \int_0^2 (x-1)\,dx + \int_0^2 \frac{dx}{x+1}$$

$$I_1 = \left[\frac{x^2}{2} - x\right]_0^2 + \Big[\ln|x+1|\Big]_0^2$$

$$I_1 = [2 - \cancel{2 - 0}] + \ln 3 - \cancel{\ln 1} = \ln 3$$

$$I_2 = (S)\int_0^{\frac{\pi}{2}} x\, d(\sin x) = \frac{\pi}{2} - 1$$

نلاحظ أن $g(x) = \sin x$ قابل للاشتقاق على $\mathbb{R}$، بالتالي

$$I_2 = \int_0^{\frac{\pi}{2}} x \cos x\,dx$$

$x \quad \cos x$

$1 \quad \sin x$

$0 \quad -\cos x$

$$= \Big[x\sin x\Big]_0^{\frac{\pi}{2}} + \Big[\cos x\Big]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi}{2}(1) - 0\right] + [0 - 1] = \frac{\pi}{2} - 1$$

$$I_3 = (S)\int_{-1}^{1} x \, d(\arctan x) = 0$$

نعلم أن $g(x) = \arctan x$ قابل للاشتقاق على $]-1,1[$

$$I_3 = \int_{-1}^{1} x \frac{1}{x^2+1} \, dx$$

نلاحظ أن $\frac{x}{x^2+1}$ تابع فردي ومتناظر على $[-1,1]$: $I_3 = 0$

$\frac{2}{11}$ نفرض أن

$$g(x) = \begin{cases} x+2 & -2 \le x \le -1 \\ 2 & -1 < x < 2 \\ x+3 & 2 \le x \le 3 \end{cases}$$

$$I = (S)\int_{-2}^{3} x \, d(g(x)) = +6$$

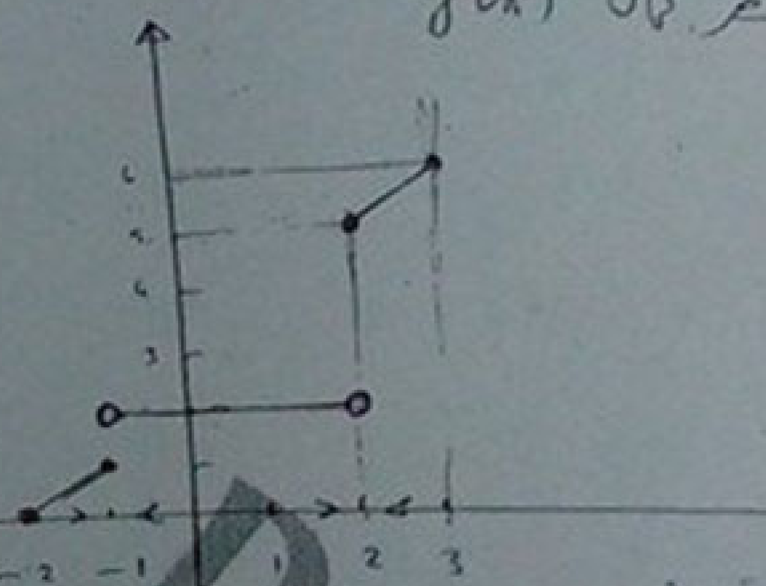
فلاحظ أن :
حيث أن $f(x)=x$
نرسم الـ $g(x)$

$$I = \int_{-2}^{-1} x(1)\,dx + \int_{-1}^{2} x(0)\,dx$$

$$+ \int_{2}^{3} x(1)\,dx +$$

$$f(-1)\left[g(-1+0) - g(-1-0)\right]$$

$$f(2)\left[g(2+0) - g(2-0)\right]$$



$$= \frac{1}{2}\left[x^2\right]_{-2}^{-1} + 0 + \frac{1}{2}\left[x^2\right]_{2}^{3} + (-1)(2-1) + (2)(5-2)$$

$$= \frac{1}{2}[1-4] + \frac{1}{2}[9-4] - 1 + 6 = -\frac{3}{2} + \frac{5}{2} + 5 = 6$$

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اذا كان
$$g(x)=\begin{cases} -1 & 0\le x\le \frac{1}{2} \\ 0 & \frac{1}{2}<x<\frac{3}{2} \\ 2 & x=\frac{3}{2} \\ -2 & \frac{3}{2}<x\le 2 \end{cases}$$

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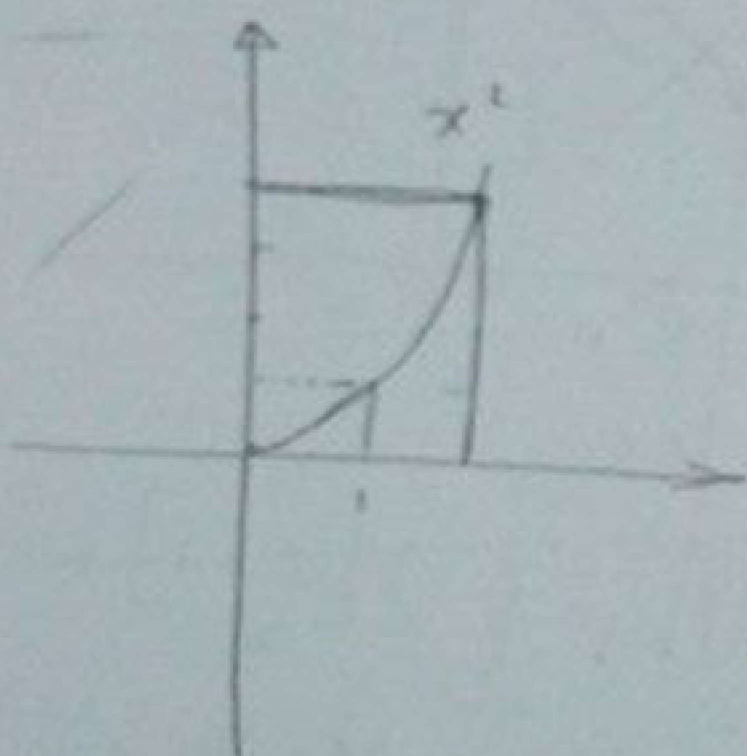
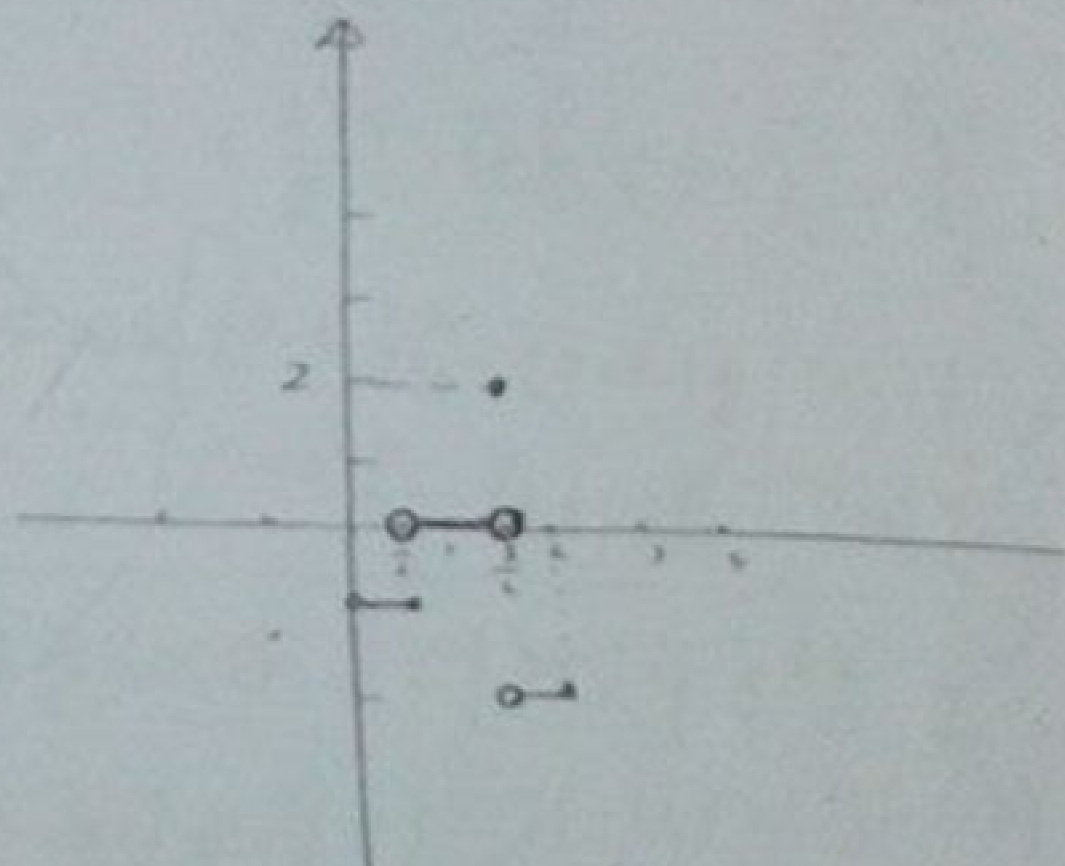
$f(x) = x^2$

$$I = (S)\int_0^2 x^2\, d(g(x)) = \frac{-17}{4}$$ اثبت ان





$$I = f\left(\tfrac{1}{2}\right)\left[g\left(\tfrac{1}{2}+0\right) - g\left(\tfrac{1}{2}-0\right)\right] + f\left(\tfrac{3}{2}\right)\left[g\left(\tfrac{3}{2}+0\right) - g\left(\tfrac{3}{2}-0\right)\right.$$

$$I = \frac{1}{4}\left[0-(-1)\right] + \frac{9}{4}\left[-2-0\right] = \frac{1}{4} - \frac{18}{4} = \frac{-17}{4}$$

4/1

$f(x)=x \qquad g(x)=\begin{cases} x+2 & -2\le x\le -1 \\ 2 & -1<x<0 \\ x^2+3 & 0\le x\le 2 \end{cases}$ بفرض

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$I=(S)\int_{-2}^{2} x \, d(g(x)) = \frac{17}{6}$ اثبت ان

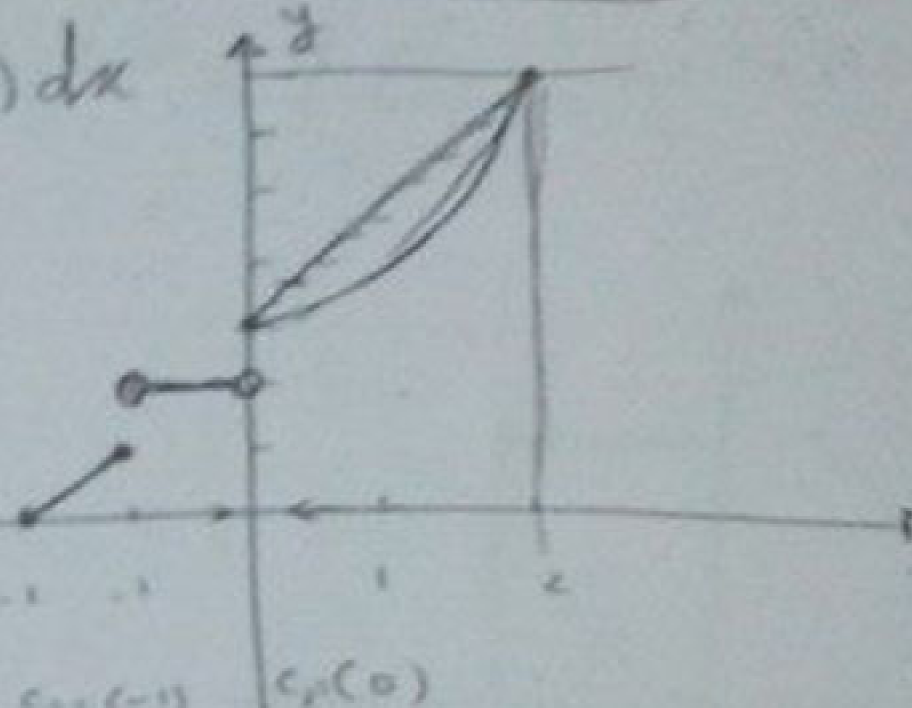
(a)

$$I=\int_{-2}^{-1} x(1)\,dx+\int_{-1}^{0} x(0)\,dx$$

$$+\int_{0}^{2} x(2x)\,dx$$

$$+f(-1)\left[g(-1+0)-g(-1-0)\right]$$

$$+\cancel{f(0)\left[g(0+0)-g(0-0)\right]}$$



$$I=\frac{1}{2}\left[x^2\right]_{-2}^{-1}+0+\frac{2}{3}\left[x^3\right]_0^2+(-1)\left[2-1\right]+$$

$$I=\frac{1}{2}\left[1-4\right]+\frac{2}{3}\left[8-0\right]-1=-\frac{3}{2}+\frac{16}{3}-1$$

$$=-\frac{5}{2}+\frac{16}{3}=\frac{-15+32}{6}=\frac{17}{6}$$

(b) $J=(S)\int_{-2}^{2} x^2 \, d(g(x)) = \frac{34}{3}$ اثبت ان

(c) $k=(S)\int_{-2}^{2} (x^2+1) \, d(g(x)) = \frac{55}{3}$

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$$J = \int_{-2}^{-1} x^2 (1)\,dx + \int_{-1}^{0} x^2 (0)\,dx + \qquad (f(x) = x^2)$$

$$\int_{0}^{2} x^2 (2x)\,dx + f(-1)\{g(-1+0) - g(-1-0)\}$$

$$+ f(0)[g(0+0) - g(0-0)]$$

$$J = \frac{1}{3}[x^3]_{-2}^{-1} + 0 + \frac{2}{4}[x^4]_{0}^{2} + 1[2-1] + 0$$

$$J = \frac{1}{3}[-1-(-8)] + \frac{1}{2}[16] + 1$$

$$J = \frac{7}{3} + \frac{9}{1} = \frac{7+27}{3} = \frac{34}{3}$$

$$f(x) = x^2+1$$

$$K = \int_{-2}^{-1} (x^2+1)(1)\,dx + \int_{-1}^{0} \cancel{(x^2+1)}(0)\,dx$$

$$+ \int_{0}^{2} (x^2+1)(2x)\,dx + f(-1)[g(-1+0) - g(-1-0)]$$

$$+ f(0)[g(0+0) - g(0-0)]$$

$$K = \left[\frac{1}{3}x^3 + x\right]_{-2}^{-1} + \left[\frac{1}{2}x^4 + x^2\right]_{0}^{2} + 2[2-1]$$

$$+ (1)[3-2]$$

$$K = \left[\left(\frac{-1}{3} - 1\right) - \left(\frac{-8}{3} - 2\right)\right] + \{(8+4) - 0\} + 2 + 1$$

$$K = \frac{4}{3} + 12 + 3 = \text{[illegible]} = 16 + \frac{1}{3} = \frac{55}{3}$$

$$\frac{7}{3} + 1$$

١) احسب التكاملات

$$I_1 = \int_0^1 x \, d\left(\ln(x^2+1)\right) \longrightarrow 2 - \frac{\pi}{2}$$

$$I_2 = \int_0^{\pi} \sin 2x \, d(x^2) \longrightarrow -\pi$$

$$I_3 = \int_{-1}^{5} \operatorname{arctg} x \, d(6x) \longrightarrow 30 \operatorname{arctg} 5 - \frac{3\pi}{2} - 3\ln$$

$$I_4 = \int_{-8}^{2} \frac{1}{2} \, d[\operatorname{ch} 2x] \longrightarrow \frac{1}{2}(\operatorname{ch}4 - \operatorname{ch}16)$$

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١) احسب التكاملين

$$I = (S)\int_{-5}^{1} f(x) \, d(g(x))$$

$$J = (S)\int_{-5}^{1} g(x) \, d(f(x))$$

حيث $f(x) = |x|$

$$g(x) = \begin{cases} x+2 & -5 \le x \le -1 \\ x^2 & -1 < x < 0 \\ 3 & x = 0 \\ \ln(x+2) & 0 < x < 1 \\ 2 & x = 1 \end{cases}$$

بين أن التكامل $\int_{-1}^{5} f(x) \, d g(x)$ غير موجود إذا علمت أن

$$f(x) = \begin{cases} 1 & x \ne 0 \\ 3 & x = 0 \end{cases}$$

$$g(x) = \begin{cases} 0 & x \ne 0 \\ -2 & x = 0 \end{cases}$$

$$I_1 = (S)\int_0^1 x \, d \ln(x^2+1) = (R)\int_0^1 \frac{2x^2}{x^2+1}\,dx$$

$$= (R)\int_0^1 \frac{2\cdot(x^2+1) - 2}{x^2+1}\,dx$$

$$= \int_0^1 \left[2 - 2\,\frac{1}{x^2+1}\right] dx = 2\,[x]_0^1 - 2\,[\arctan x]_0^1$$

$$= 2 - 2\left[\frac{\pi}{4} - 0\right] = 2 - \frac{\pi}{2}$$

$$I_2 = (S)\int_0^{\pi} \sin 2x \; d(x^2) \qquad : \quad d(x^2) = 2x\,dx$$

$$= (R)\; 2\int_0^{\pi} x \sin 2x \; dx$$

$$\begin{array}{lcl} x & + & \sin 2x \\ 1 & - & -\frac{1}{2}\cos 2x \\ 0 & & -\frac{1}{4}\sin 2x \end{array}$$

$$I_2 = 2\left\{ -\frac{1}{2}\left[x \cos 2x\right]_0^{\pi} + \frac{1}{4}\left[\sin 2x\right]_0^{\pi} \right\}$$

$$= 2\left\{ -\frac{1}{2}\left[\pi - 0\right] + \frac{1}{4}\left[0 - 0\right] \right\} = -\pi$$

$$I_3 = \int_{-1}^{5} \arctan x \; d(6x) = 6\int_{-1}^{5} \underbrace{\arctan x}_{u}\; \underbrace{dx}_{dv}$$

$$u = \arctan x \Rightarrow du = \frac{1}{1+x^2}\,dx$$

$$dv = dx \Rightarrow v = x$$

$$I_3 = 6\left[x \arctan x\right]_{-1}^{5} - 3\int_{-1}^{5} \frac{2x}{1+x^2}\,dx$$

$$= 6\left[5 \arctan 5 + \left(-\frac{\pi}{4}\right)\right] - 3\left[\ln(1+x^2)\right]_{-1}^{5}$$

$$= 30 \arctan 5 - \frac{3\pi}{2} - 3\left[\ln 26 - \ln 2\right] = 30 \arctan 5 - \frac{3\pi}{2} - 3\ln 13$$

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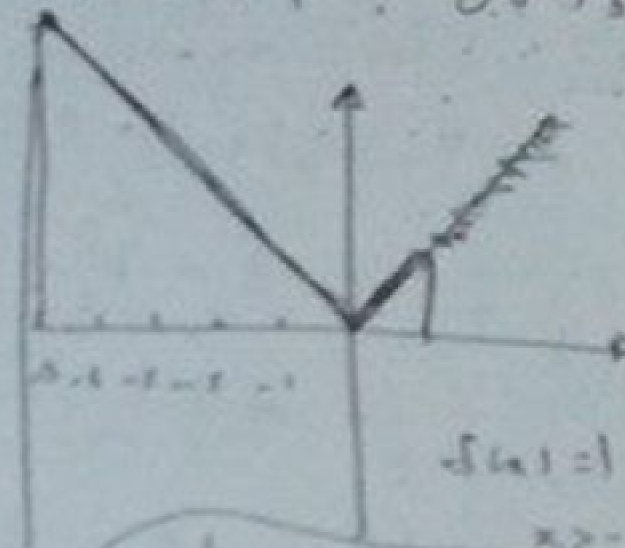
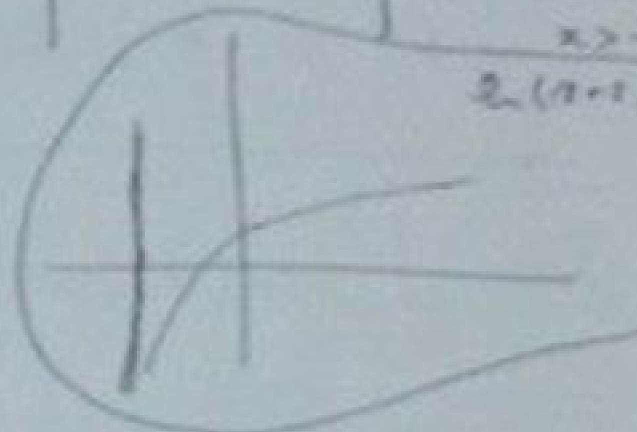
احسب 1) $\int_{-5}^{1} f(x)\, d g(x)$ (s) , $\int_{-5}^{1} g(x)\, d f(x)$ (s)

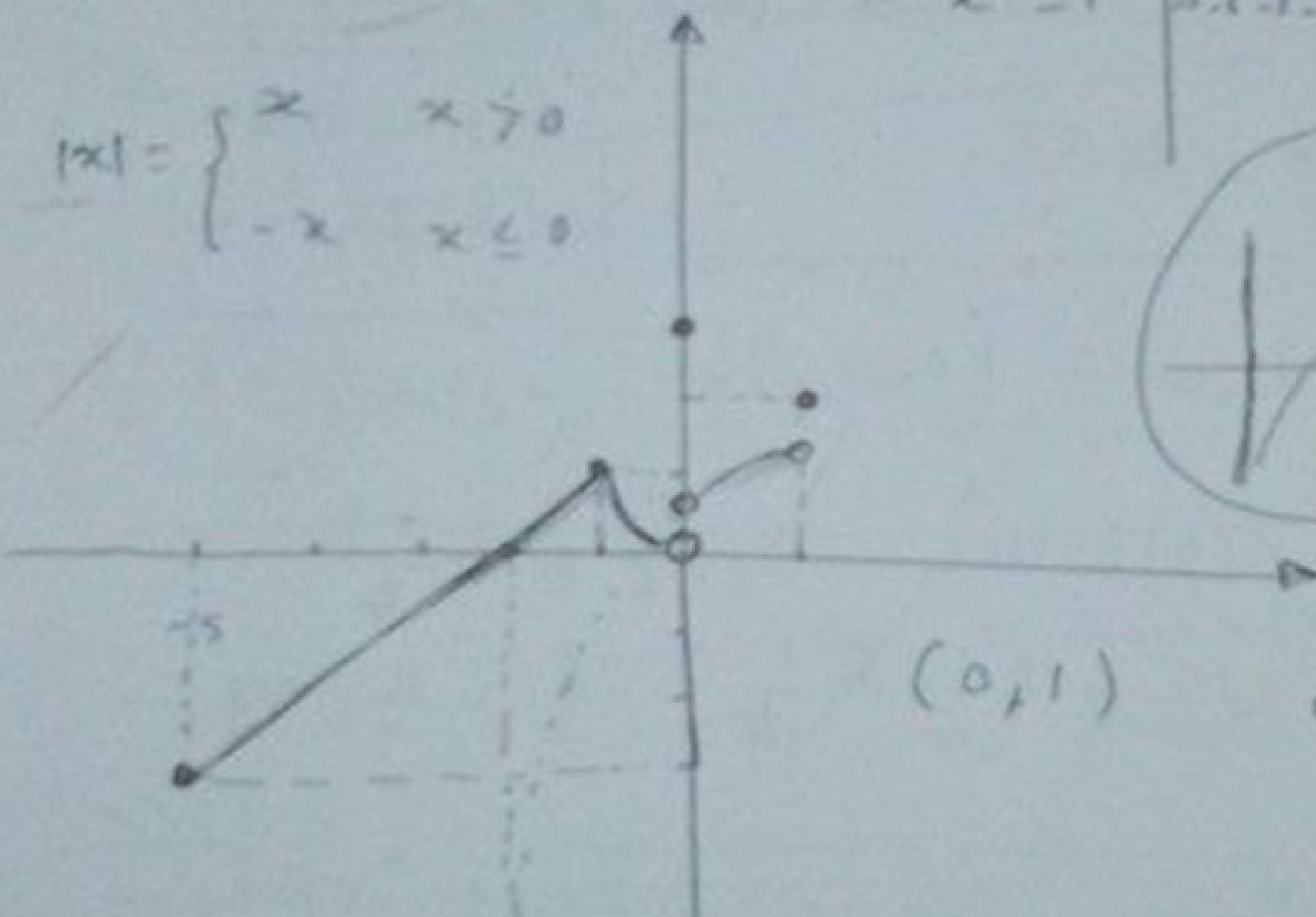
$$g(x) = \begin{cases} x+2 & -5 \le x \le -1 \\ x^2 & -1 < x < 0 \\ 3 & x = 0 \\ \ln(x+2) & 0 < x < 1 \\ 2 & x = 1 \end{cases}$$

, $f(x) = |x|$. حيث





$$|x| = \begin{cases} x & x \ge 0 \\ -x & x \le 0 \end{cases}$$



$(0,1)$ لماذا نقطة ؟

$$g'(x) = \begin{cases} 1 & -5 < x < -1 \\ 2x & -1 < x < 0 \\ \frac{1}{x+2} & 0 < x < 1 \end{cases}$$

كل
معرفة
في
$[-5,1]$

2) $\int_{-5}^{1} f(x)\, d g(x) = \int_{-5}^{-1} -x\,(1)\, dx - \int_{-1}^{0} x(2x)\, dx$

$+ \int_{0}^{1} x \frac{1}{x+2}\, dx + f(0)\left[g(0+0) - g(0-0)\right] + f(1)\left[g(1+0) - g(1-0)\right]$

$$I = \int_{-5}^{1} f(x)\, dg(x) = -\frac{1}{2}\left[x^2\right]_{-5}^{-1} - \frac{2}{3}\left[x^3\right]_{-1}^{0}$$

$$+ \left[x\right]_0^1 - 2\cdot\left[\ln(x+2)\right]_0^1 + 0 + 1\left[2 - \ln(3)\right]$$

$$I = -\frac{1}{2}\left[1 - 25\right] - \frac{2}{3}\left[0 + 1\right] + \left[1 - 0\right] - 2\left[\ln 3 - \ln 2\right]$$

$$+ 2 - \ln 3$$

$$= 12 - \frac{2}{3} + 1 - 2\ln 3 + 2\ln 2 + 2 - \ln 3$$

$$= 15 - \frac{2}{3} - \ln\frac{27}{4}$$

$$\boxed{\begin{array}{l}\ln 4 - \ln 27 \\ -(\ln 27 - \ln 4)\end{array}}$$

$$J = \int_{-5}^{1} g(x)\, df(x) = \left[f(x)\cdot g(x)\right]_{-5}^{1} - 15 + \frac{2}{3} + \ln\frac{27}{4}$$

$$= \left[(1)(2) - (5)(-3)\right] - 15 + \frac{2}{3} + \ln\frac{27}{4}$$

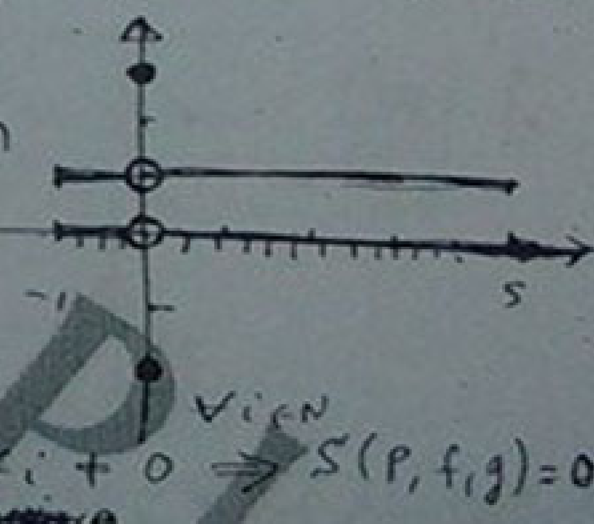
$$= 2 + \cancel{15} - \cancel{15} + \frac{2}{3} + \ln\frac{27}{4} = \frac{8}{3} + \ln(27/4)$$

تمرين:

بين أن $\int_{-1}^{5} f\, dg$ غير موجود حيث

$$g(x) = \begin{cases} 1 & x \neq 0 \\ 3 & x = 0 \end{cases}$$

$$f(x) = \begin{cases} 0 & x \neq 0 \\ -2 & x = 0 \end{cases}$$



نأخذ التجزئة

$$P = \{-1 = x_0 \underset{t_1}{\underline{<x_1}} \underset{t_2}{\underline{<x_2}} \cdots \underset{t_i}{\underline{<x_{i-1} < x_i}} \underset{t_{i+1}}{\underline{<x_{i+1}}} \underset{t_n}{\underline{<x_n = 5}}\}$$

$$S(P,f,g) = \sum_{k=1}^{n} f(t_k)\left[g(x_k) - g(x_{k-1})\right]$$

$$= 0 + 0 + \cdots + 0 + f(t_i)\left[g(0) - g(x_{i-1})\right]$$

$$+ f(t_{i+1})\left[g(x_{i+1}) - g(0)\right] + 0 + 0 \cdots$$

$$= f(t_i)\left[-2 - 0\right] + f(t_{i+1})\left[0 - (-2)\right] = 2\left[f(t_{i+1}) - f(t_i)\right] = 2\begin{cases} 1-3 & t_i = 0 \\ 3-1 & t_{i+1} = 0 \end{cases}$$

$\forall i \in N$

① $x_i \neq 0 \Rightarrow S(P,f,g) = 0$

② $x_i = 0 \Rightarrow t_i < 0$, $t_{i+1} > 0$, $t_i = 0$

س ٥ : بين أي من الدوال ذات تغير محدود وأوجد التغير الكلي في كل من الحالات التالية

$f(x) = x - |x|$ حيث $x \in [-5, 5]$

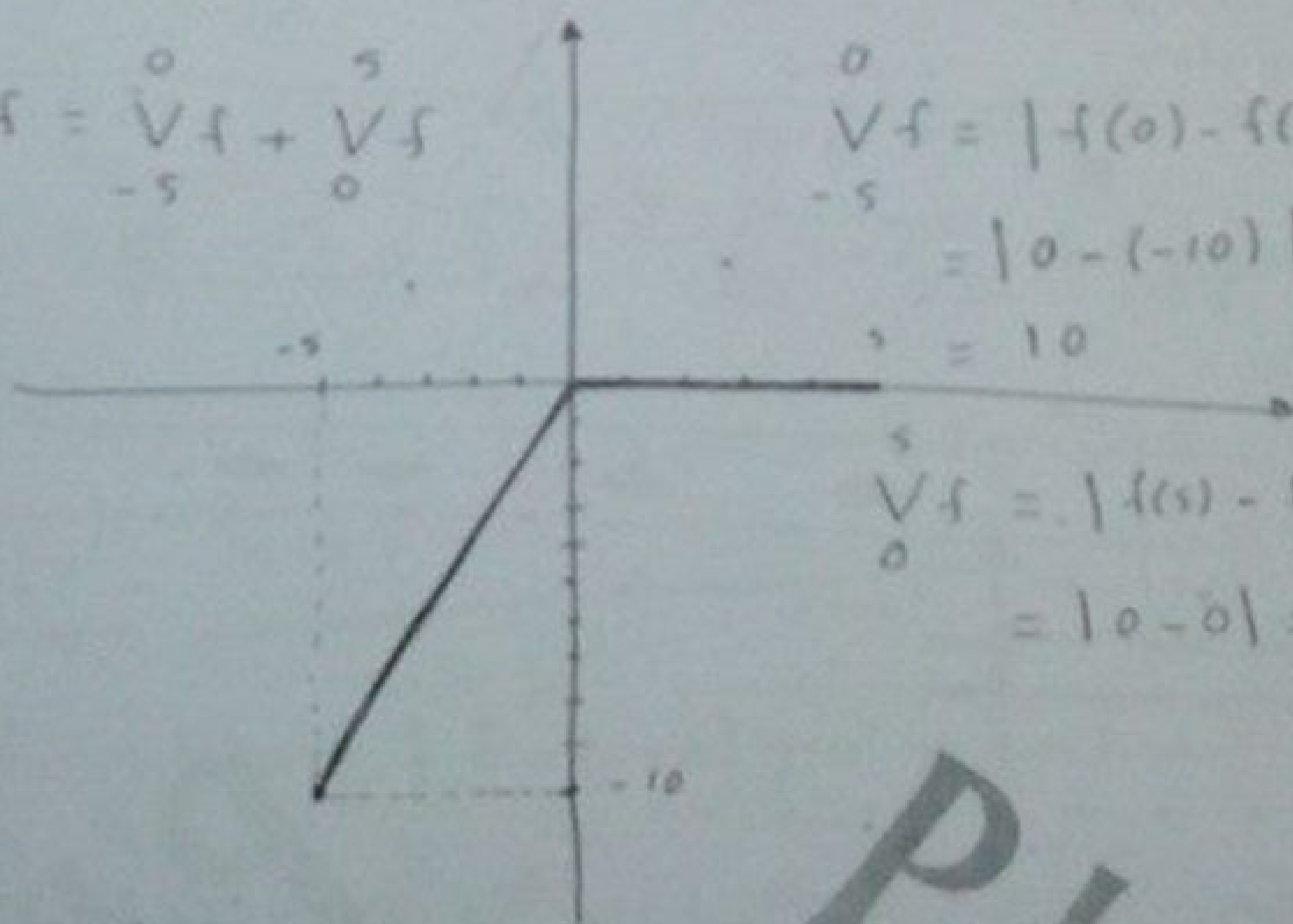
ج ٥

$$|x| = \begin{cases} x & 5 \geq x \geq 0 \\ -x & -5 \leq x \leq 0 \end{cases}$$

$$f(x) = \begin{cases} x - x & 0 \leq x \leq 5 \\ x - (-x) & -5 \leq x \leq 0 \end{cases}$$

$$f(x) = \begin{cases} 0 & 0 \leq x \leq 5 & \text{متزايدة} \\ 2x & -5 \leq x \leq 0 & \text{متزايدة} \end{cases}$$

$$\overset{5}{\underset{-5}{V}} f = \overset{0}{\underset{-5}{V}} f + \overset{5}{\underset{0}{V}} f$$



$$\overset{0}{\underset{-5}{V}} f = |f(0) - f(-5)|$$
$$= |0 - (-10)|$$
$$= 10$$

$$\overset{5}{\underset{0}{V}} f = |f(5) - f(0)|$$
$$= |0 - 0| = 0$$

$$\overset{5}{\underset{-5}{V}} f = \overset{0}{\underset{-5}{V}} f + \overset{5}{\underset{0}{V}} f = 10 + 0 = 10 < \infty$$

f ذات تغير محدود والتغير الكلي 10

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بيّن أن f د.ت.م على $[0,1]$ ثم

$$f(x) = x^2 + \frac{1}{x+1}$$

$$= x^2 - \left(-\frac{1}{x+1}\right)$$

نلاحظ أن x^2 دالة متزايدة على $[0,1]$. $\left[f'(x) = 2x \geq 0\right]$

$-\frac{1}{x+1}$ = = على $[0,1]$. $\left[f'(x) = \frac{1}{(x+1)^2} \geq 0\right]$

بالتالي الدالة كتبت على شكل فرق دالتين متزايدتين

بالتالي f د.ت.م على $[0,1]$.

بين أن f د. م. ت على $[a,b]$ حيث $a<0,\ b>0$

$$f(x) = x - e^{x}$$

$$f'(x) = 1 - e^{x}$$

$$f'(x) = 0 \Rightarrow 1 - e^{x} = 0 \Rightarrow e^{x} = 1$$

$$x = 0 \Rightarrow f(0) = -1$$

x	$-\infty$	a		0		b	$+\infty$
$f'(x)$		$+$	$+$	0	$-$		
$f(x)$	$-\infty$	متزايدة $\nearrow$		-1	متناقصة $\searrow$		$-\infty$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} (x - e^{x}) = -\infty - 0 = -\infty$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} (x - e^{x}) = \underbrace{+\infty}_{\text{ببطء}} - \underbrace{\infty}_{\text{بسرعة كبيرة}} = -\infty$$

$$\overset{b}{\underset{a}{V}} f = \overset{0}{\underset{a}{V}} f + \overset{b}{\underset{0}{V}} f = |f(0) - f(a)| + |f(0) - f(b)|$$

$$= |-1 - (a - e^{a})| + |-1 - (b - e^{b})|$$

$$= e^{a} - a - 1 + e^{b} - b - 1 = e^{a} + e^{b} - (a + b + 2)$$

طريقة أخرى

$$\overset{b}{\underset{a}{V}} f = \int_a^b |1 - e^{x}|\, dx = \int_a^0 (1 - e^{x})\, dx - \int_0^b (1 - e^{x})\, dx$$

$$= [x - e^{x}]_a^0 - [x - e^{x}]_0^b$$

$$= [(-1) - (a - e^{a})] - [(b - e^{b}) - (-1)] = e^{a} - a - 1 - (b + 1 - e^{b})$$

$$= e^{a} + e^{b} - (a + b + 2)$$

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س٥ ! هل f د . ت . م على $[0,2\pi]$ حيث

$$f(x) = \begin{cases} x \sin\frac{\pi}{x} & 0 < x \le 2\pi \\ 0 & x = 0 \end{cases}$$

نبين أن f دالة ليست د . ت . م على $[0,2]$ وبالتالي ليست د . ت . م على $[0,2\pi]$.

وذلك بأخذ التجزئة التالية

$$x_0 = 0 < \frac{2}{2n+1} < \frac{2}{2n-1} < \cdots \frac{2}{5} < \frac{2}{3} < \frac{2}{1} = x_n$$

$$V(f,P) = \left|f\left(\tfrac{2}{2n+1}\right) - f(0)\right| + \left|f\left(\tfrac{2}{2n-1}\right) - f\left(\tfrac{2}{2n+1}\right)\right| + \cdots$$

$$\cdots + \left|f\left(\tfrac{2}{3}\right) - f\left(\tfrac{2}{5}\right)\right| + \left|f(2) - f\left(\tfrac{2}{3}\right)\right|$$

$$= \left|\frac{2}{2n+1}\sin\left(\frac{2n+1}{2}\pi\right) - 0\right| + \left|\frac{2}{2n-1}\sin\frac{2n-1}{2}\pi - \frac{2}{2n+1}\sin\frac{2n+1}{2}\pi\right|$$

$$\cdots + \left|\frac{2}{3}\sin\frac{3}{2}\pi - \frac{2}{5}\sin\frac{5}{2}\pi\right| + \left|2\sin\frac{\pi}{2} - \frac{2}{3}\sin\frac{3}{2}\pi\right|$$

$$= \frac{2}{2n+1} + \frac{2}{2n-1} + \frac{2}{2n+1} + \cdots \frac{2}{3} + \frac{2}{5} + 2 + \frac{2}{3}$$

$$= \frac{4}{2n+1} + \frac{4}{2n-1} + \cdots \frac{4}{5} + \frac{4}{3} + 2 = \left(\sum_{k=0}^{n} \frac{4}{2n+1}\right) - 2$$

$$\overset{2}{\underset{0}{V}} f = \lim_{n\to\infty}\left[\left(\sum_{k=0}^{n} \frac{4}{2n+1}\right) - 2\right] = \infty$$

بما أن $\overset{2}{\underset{0}{V}} f = +\infty$ ، الدالة f ليست ذات
تغير محدود

$f(x) = \cos^2 x \qquad x \in [0, \pi]$

$f(x) = 1 - \sin^2 x$

$f_1(x) = 1$

$f_2(x) = \sin^2 x$

$f_2'(x) = 2 \sin x \cos x = \sin 2x$

$f_2'(x) = 0 \Rightarrow \sin 2x = 0 \Rightarrow 2x = \pi k$

$x = \frac{\pi}{2} k \qquad \begin{cases} k = 0 \Rightarrow x = 0 \\ k = 1 \Rightarrow x = \frac{\pi}{2} \\ k = 2 \Rightarrow x = \pi \end{cases}$

x	0		$\frac{\pi}{2}$		π
$f_2'(x)$		$+$	0	$-$	
$f(x)$	0	$\nearrow$	1	$\searrow$	0

طريقة ١ $\quad f(x) = \begin{cases} 1 - \sin^2 x & : x \in [0, \frac{\pi}{2}] \\ \cos^2 x - 0 & : x \in [\frac{\pi}{2}, \pi] \end{cases}$

طريقة ٢ $\quad f(x) = x - (x - \cos^2 x) \qquad x \in [0, \pi]$

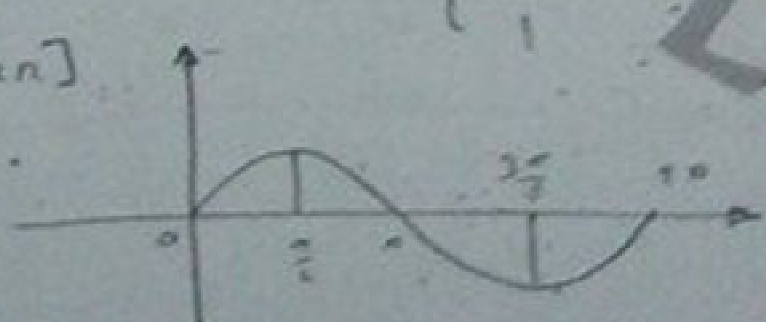
$f(x) = \sin x \Rightarrow f(x) = x - (x - \sin x)$

تمرين ١٠ $\qquad x \in [0, 2\pi]$

$f(x) = \sin x = \varphi(x) - \psi(x)$

طريقة

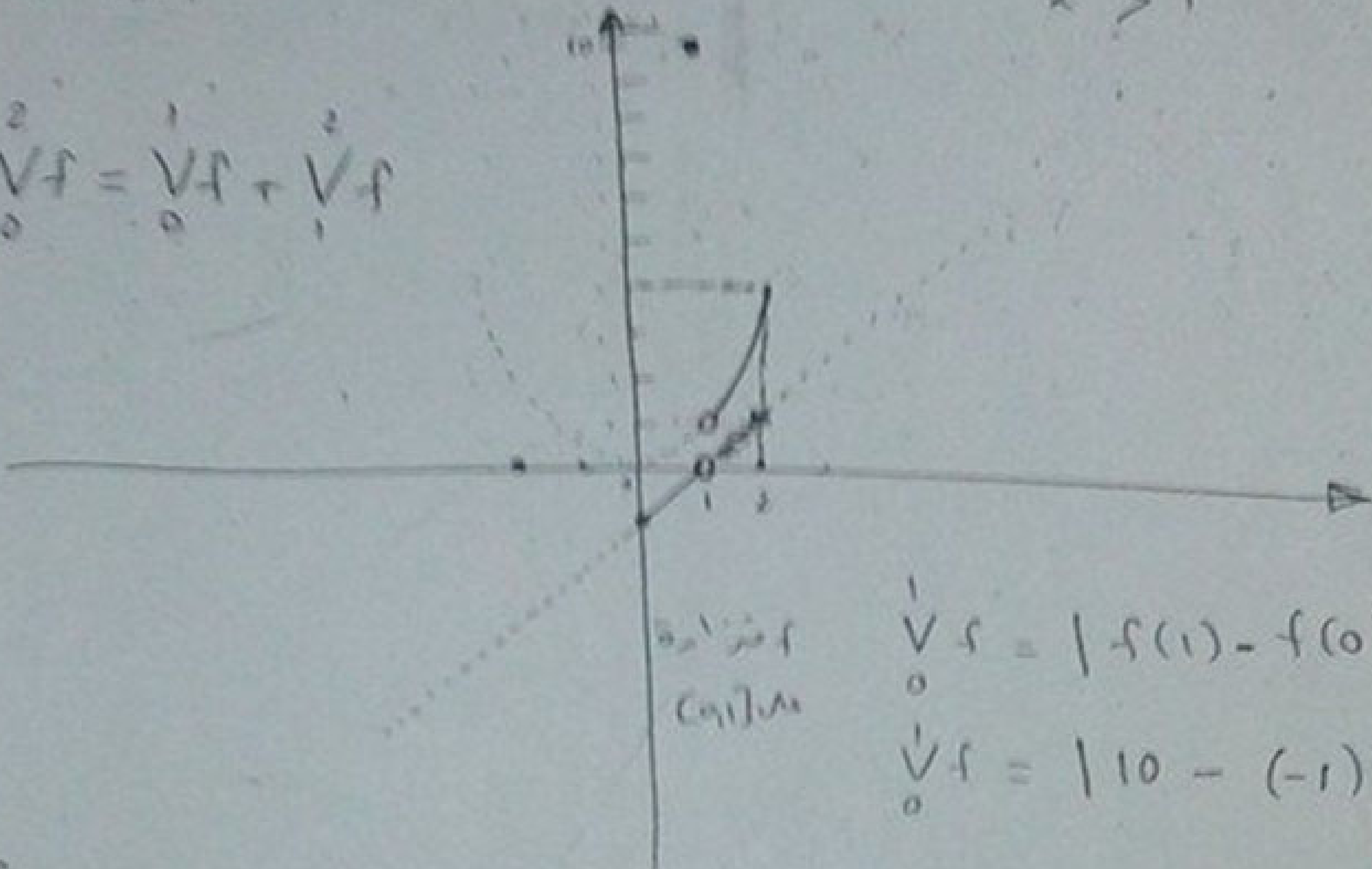
$\varphi(x) = \begin{cases} \sin x & x \in [0, \frac{\pi}{2}] \\ 1 & [\frac{\pi}{2}, 3\frac{\pi}{2}] \\ \sin x + 1 & [3\frac{\pi}{2}, 2\pi] \end{cases} \qquad \psi(x) = \begin{cases} 0 & x \in [0, \frac{\pi}{2}] \\ \sin x & x \in [\frac{\pi}{2}, 3\frac{\pi}{2}] \\ 1 & x \in [3\frac{\pi}{2}, 2\pi] \end{cases}$



٧/ بيّن أن التغير الكلي للدالة في 25 على [0,2] .

$$f(x) = \begin{cases} x-1 & x<1 \\ 10 & x=1 \\ x^2 & x>1 \end{cases}$$

$$\overset{2}{\underset{0}{V}} f = \overset{1}{\underset{0}{V}} f + \overset{2}{\underset{1}{V}} f$$



$$\overset{1}{\underset{0}{V}} f = |f(1) - f(0)|$$

$$\overset{1}{\underset{0}{V}} f = |10 - (-1)| = 11$$

$$\overset{2}{\underset{1}{V}} f = \sup_{P \in \mathbb{P}[1,2]} V(f,P) \qquad P = \{1 = x_0 < x_1 < x_2 \cdots < x_n = 2\}$$

$$V(f,P) = |f(x_1) - f(x_0)| + |f(x_1) - f(x_2)| + \cdots \; |f(x_n) - f(x_{n-1})|$$

$$= |x_1^2 - 10| + |x_1^2 - x_2^2| + \cdots \; |x_n^2 - x_{n-1}^2|$$

$$= 10 - \cancel{x_1^2} + \cancel{x_2^2} - \cancel{x_1^2} + \cancel{x_3^2} - \cancel{x_2^2} + \cdots + \underbrace{x_n^2}_{4} - \cancel{x_{n-1}^2}$$

$$V(f,P) = 10 - 2x_1^2 + 4 = 14 - 2x_1^2$$

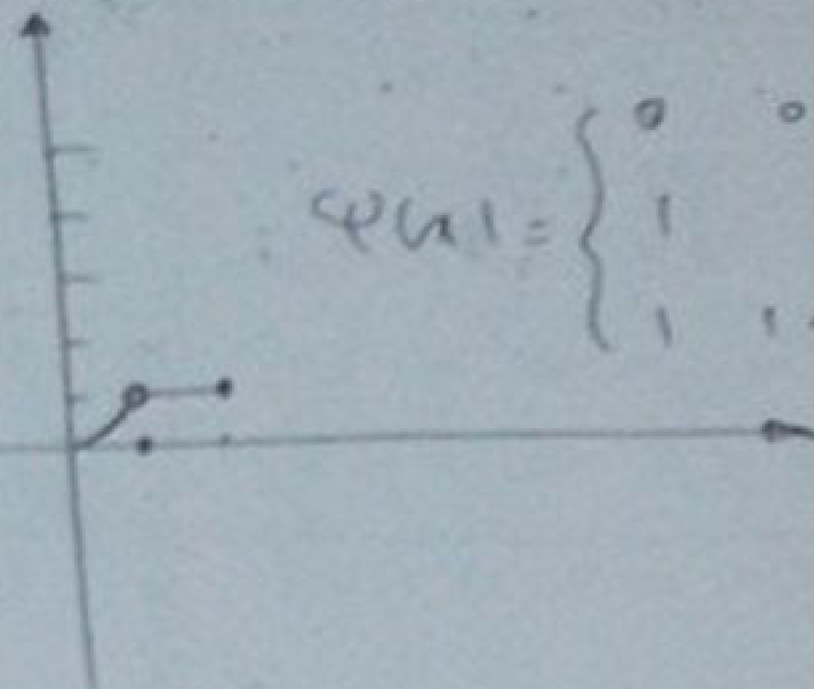
$$\overset{2}{\underset{1}{V}} f = 12$$

$$\Longrightarrow \overset{2}{\underset{0}{V}} f = 11 + 12 = 23 .$$

c) $$f(x)=\begin{cases} x^2 & 0\le x<1 \\ 0 & x=1 \\ 1 & 1<x\le 2 \end{cases}$$

$f(x)=\varphi(x)-\psi(x)$

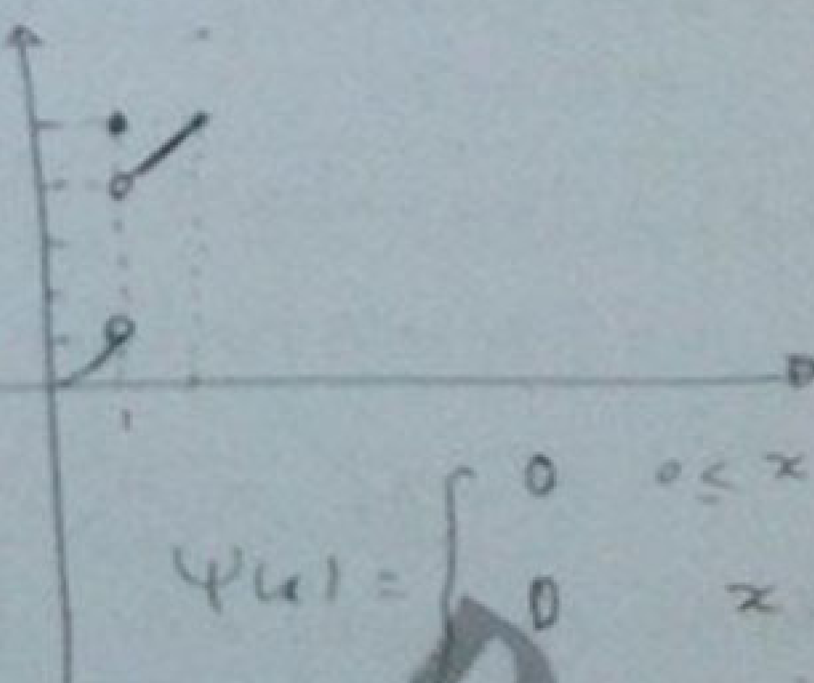
$$\varphi(x)=\begin{cases} x^2 & 0\le x<1 \\ 1 & x=1 \\ 2 & 1<x\le 2 \end{cases}$$



$$\psi(x)=\begin{cases} 0 & 0\le x<1 \\ 1 & x=1 \\ 1 & 1<x\le 2 \end{cases}$$

D) $$f(x)=\begin{cases} x & 0\le x<1 \\ 5 & x=1 \\ x+3 & 1<x\le 2 \end{cases}$$

$f(x)=\varphi(x)-\psi(x)$



$$\varphi(x)=\begin{cases} x^2 & 0\le x<1 \\ 5 & x=1 \\ x+5 & 1<x\le 2 \end{cases}$$

$$\psi(x)=\begin{cases} 0 & 0\le x<1 \\ 0 & x=1 \\ 2 & 1<x\le 2 \end{cases}$$