Syria Math

تحليل ١

الاستاذة : علا أسبر

المحاضرة : الرابعة عملي

إعداد : رهف + رسمية + شهيناز

$$\sum_{n=1}^{\infty} \frac{1}{(3n+1)(3n+4)} = \frac{1}{3}\left[\frac{1}{(3n+1)} - \frac{1}{(3n+4)}\right]$$

$$b_n = \frac{1}{(3n+1)} \quad , \quad b_{n+1} = \frac{1}{3n+4} \qquad b_1 = \frac{1}{4}$$

$$l = \lim_{n\to\infty} b_n = 0$$

$$\Sigma = \frac{1}{3}\left(\frac{1}{4} - 0\right) = \frac{1}{12}$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+5)} = \frac{1}{6}\left[\frac{1}{2n-1} - \frac{1}{2n+5}\right]$$

$$b_n = \frac{1}{2n-1} \quad , \quad b_{n+1} = \frac{1}{2n+1}$$

$$\frac{1}{\quad} = \frac{1}{6}\left\{\frac{1}{2n-1} - \frac{1}{(2n+1)} + \frac{1}{2n+1} - \frac{1}{2n+5}\right]$$

$$c_n = \frac{1}{2n+1} \quad , \quad c_{n+1} = \frac{1}{2n+3}$$

$$\frac{1}{)\;(\;)} = \frac{1}{6}\left[\overbrace{\frac{1}{2n-1} - \frac{1}{2n+1}}^{b_n \quad b_{n+1}} + \overbrace{\frac{1}{2n+1} - \frac{1}{2n+3}}^{c_n \quad c_{n+1}}\right.$$

$$\overset{d_n}{+\frac{1}{2n+3}} \quad \overset{d_{n+1}}{-\frac{1}{2n+5}} \}$$

$d_n = \frac{1}{2n+3}$, $d_{n+1} = \frac{1}{2n+5}$

$b_1 = 1$ $\qquad l_1 = 0$

$b_2 = \frac{1}{3}$ $\qquad l_2 = 0$

$d_1 = \frac{1}{5}$ $\qquad l_3 = 0$

$$S = \frac{1}{6}\{(1-0)+(\frac{1}{3}-0)+(\frac{1}{5}-0)\}$$

$$= \frac{1}{6}\{1+\frac{1}{3}+\frac{1}{5}\} = \frac{1}{6}\left(\frac{15+5+3}{15}\right) = \boxed{\frac{23}{90}}$$

المتسلسلة الهندسية: $\sum_{n=0}^{\infty} ar^n$ $\quad |r|<1$ متقاربة $\frac{a}{1-r}$

$|r| \geq 1$ متباعدة. $\quad \sum_{n=0}^{\infty} 2.3^n \quad |3| = 3 > 1$ متباعدة.

$$\sum_{n=0}^{\infty} 5\left(-\frac{1}{3}\right)^n \Rightarrow \left|-\frac{1}{3}\right| = \frac{1}{3} < 1$$ متقاربة

مجموعها: $\frac{5}{1+\frac{1}{3}} = \frac{5}{\frac{4}{3}} = \frac{15}{4}$

$$\sum_{n=1}^{\infty} 5\left(\frac{-1}{3}\right)^n = -5+5+\sum_{n=1}^{\infty} 5\left(\frac{-1}{3}\right)^n$$

$$= -5+\sum_{n=0}^{\infty} 5\left(\frac{-1}{3}\right)^n = -5+\frac{15}{4} = -\frac{5}{4}$$

$$\sum_{n=0}^{\infty} \frac{1}{n!} = e$$

$$\sum_{n=1}^{\infty} \frac{1}{n!} = -1+1+\sum_{n=1}^{\infty} \frac{1}{n!}$$

$$= -1+\sum_{n=0}^{\infty} \frac{1}{n!} = e-1$$

$$\sum_{n=2}^{\infty} \frac{1}{n!} = -1-1+1+1+\sum_{n=2}^{\infty} \frac{1}{n!}$$

$$= -2+\sum_{n=0}^{\infty} \frac{1}{n!} = e-2$$

$$\sum_{n=1}^{\infty} \frac{n}{n!} = \sum_{n=1}^{\infty} \frac{1}{(n-1)!} = e$$

$$\sum_{n=0}^{\infty} \frac{1}{k!} = e$$ من هذا الشكل

Alamal

$$\sum_{n=1}^{\infty} \frac{n^2}{n!} = \sum_{n=1}^{\infty} \frac{n}{(n-1)!}$$

$$= \sum_{n=1}^{\infty} \frac{n-1+1}{(n-1)!} = \sum_{n=1}^{\infty} \left[\frac{n-1}{(n-1)!} + \frac{1}{(n-1)!} \right]$$

$$\sum_{n=1}^{\infty} \frac{(n-1)}{(n-1)!} = 0 + \sum_{n=2}^{\infty} \frac{(n-1)}{(n-1)!}$$

$$= \sum_{n=2}^{\infty} \frac{1}{(n-2)!} = e$$

$$\sum_{n=1}^{\infty} \frac{1}{(n-1)!} = e \Rightarrow \sum_{n=1}^{\infty} \frac{n^2}{n!} = 2e$$

مبرهنة:

$$\sum (a_n + b_n) \rightarrow a+b \quad \left\{ \begin{array}{l} \sum a_n = a \\ \sum b_n = b \end{array} \right\}$$

$$\sum_{n=1}^{\infty} \frac{n^3}{n!} = \sum_{n=1}^{\infty} \frac{n^2}{(n-1)!} = \sum_{n=1} \frac{n^2-1+1}{(n-1)!}$$

$$\sum_{n=1}^{\infty} \frac{(n^2-1)}{(n-1)!} + \frac{1}{(n-1)!}$$

$$\sum_{n=1}^{\infty} \frac{n^2-1}{(n-1)!} = \sum_{n=2}^{\infty} \frac{(n-1)(n+1)}{(n-1)\,(n-2)!}$$

(5

$$\sum_{n=2}^{\infty} \frac{(n+1)}{(n-2)!} = \sum_{n=2}^{\infty} \frac{n-2+2+1}{(n-2)!}$$

$$= \sum_{n=2}^{\infty}\left[\frac{n-2}{(n-2)!} + \frac{3}{(n-2)!}\right]$$

$$\sum_{n=2}^{\infty} \frac{3}{(n-2)!} = 3e$$

$$\sum_{n=2}^{\infty} \frac{n-2}{(n-2)!} = \sum_{n=3}^{\infty} \frac{(n-2)}{(n-2)!} = \sum_{n=3}^{\infty} \frac{1}{(n-3)!} = e$$

$$\sum_{n=1}^{\infty} \frac{n^2-1}{(n-1)!} = 4e$$

$$\sum_{n=1}^{\infty} \frac{n^3}{n!} = 4e + e = 5e.$$