تحليل ا

الدكتور : نايف طلي

المحاضرة : الخامسة عشرة

الدوال الأساسية

1) $f: \mathbb{R} \to \mathbb{R} ; y = x - 1$

2) $f: \mathbb{R} \to \mathbb{R}^{+*} ; y = e^x$

3) $f: \mathbb{R} \to \mathbb{R}^{+*} ; y = a^x \quad ; a > 0 , a \neq 1$

4) $f: \mathbb{R} \to \mathbb{R}^{+} ; y = x^2$

5) $f: \mathbb{R} \to [-1, 1] ; y = \sin x$

6) $f: \mathbb{R} \to [-1, 1] ; y = \cos x$

7) $f:]-\frac{\pi}{2}, \frac{\pi}{2}[\to \mathbb{R} ; y = \tan x$

8) $f:]0, \pi[\to \mathbb{R} ; y = \cot an\, x$

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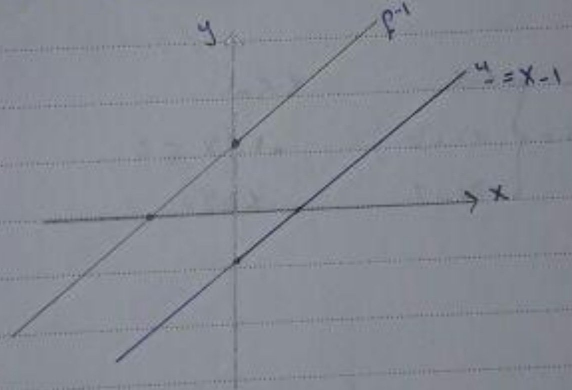
((1))

* $f: \mathbb{R} \to \mathbb{R} ; y = x - 1$

$x = y - 1$

$y = x + 1$

$f^{-1}: \mathbb{R} \to \mathbb{R} ; f^{-1}(x) = x + 1$



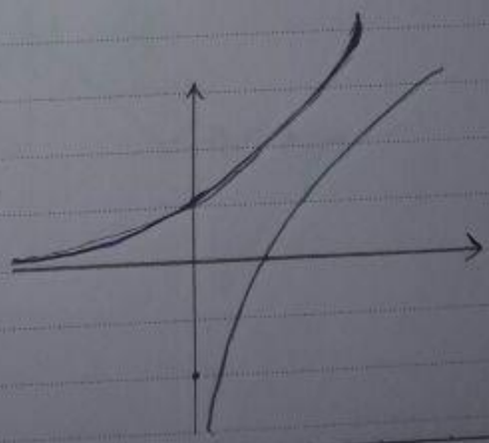
* $f: \mathbb{R} \to \mathbb{R}^{++} ; y = e^x$

$\ln y = x \ln e$

$\ln y = x$

$y = \ln x$

$f^{-1}: \mathbb{R}^{+}_{*} \to \mathbb{R} ; f^{-1}(x) = \ln x$



Syria Math

* $f: \mathbb{R} \to \mathbb{R}^{+*} : y = a^x$

$\log_a y = \log_a a^x = x$

$y = \log_a x$

$8 = 2^3 \Leftrightarrow \log 8 = 3$

$100 = 10^2 \Leftrightarrow \log_{10}^2 100 = 2$

$5 = 5^1 \Leftrightarrow \log_5^1 5 = 1$

* $f^{-1}: \mathbb{R} \to \mathbb{R} : f^{-1}(x) = \log_a x$

$\log_a x = \frac{\ln x}{\ln a}$

$y = \log_a x \Leftrightarrow x = a^y$

$\ln x = \ln a^y = y \ln a$

$y = \frac{\ln x}{\ln a}$

((< 1))

* $f: \mathbb{R} \to \mathbb{R}^+ : y = x^2$

$\sqrt{y} = |x|$

$x = \sqrt{y}$ si $x = -\sqrt{y}$

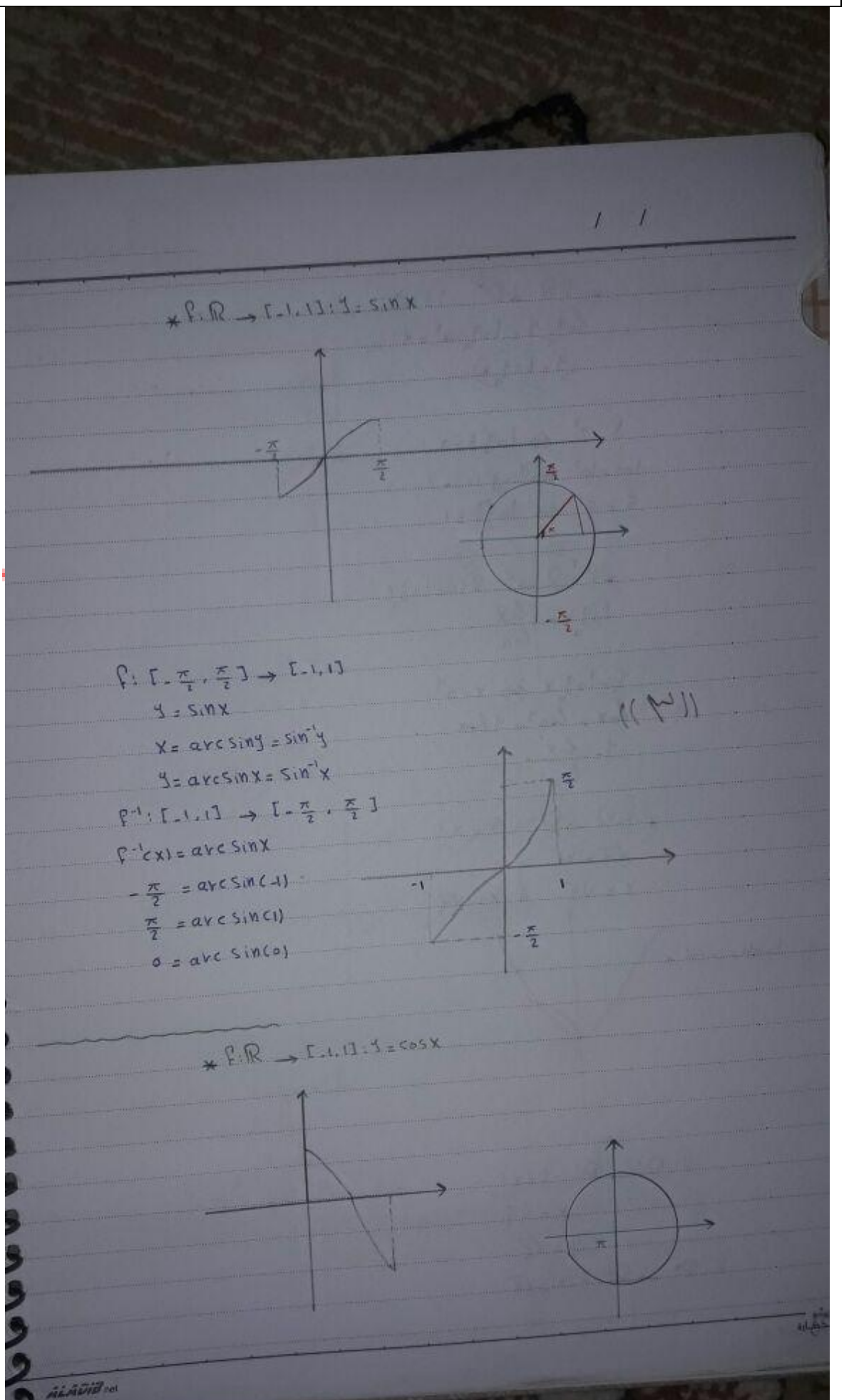
ليس دالة ←

$f: \mathbb{R}^+ \to \mathbb{R}^+ : y = x^2$

$x = \sqrt{y}$

$y = \sqrt{x}$

$f^{-1}: \mathbb{R}^+ \to \mathbb{R}^+ : f^{-1}(x) = \sqrt{x}$


* f: R → [-1, 1] ; y = sin x
f: [-π/2 , π/2] → [-1, 1]
y = sin x
x = arc sin y = sin⁻¹ y
y = arc sin x = sin⁻¹ x
f⁻¹: [-1, 1] → [-π/2 , π/2]
f⁻¹(x) = arc sin x
-π/2 = arc sin(-1)
π/2 = arc sin(1)
0 = arc sin(0)
* f: R → [-1, 1] ; y = cos x

Syria Math

$f: [0, \pi] \to [-1, 1]$

$y = \cos x$

$x = \arccos y = \cos^{-1} y$

$y = \arccos x$

$f^{-1}: [-1, 1] \to [0, \pi]$

$f^{-1}(x) = \arccos x$

$\pi = \arccos(-1)$

$\frac{\pi}{2} = \arccos(0)$

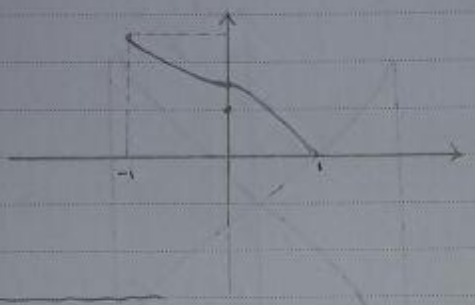
$0 = \arccos(1)$

تذكر دائما

$\arccos(-1) = y$

$-1 = \cos y$

$y = \pi$



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$* f:]-\frac{\pi}{2}, \frac{\pi}{2}[\to \mathbb{R}: y = \tan x$

$y = \tan x \to tgx$

$x = \arctan y = \tan^{-1} y$

$y = \arctan x = \tan^{-1} x$

$f^{-1}: \mathbb{R} \to]-\frac{\pi}{2}, \frac{\pi}{2}[: f^{-1}(x) = \arctan x$

$\arctan(-\infty) \to -\frac{\pi}{2}$

$\arctan(+\infty) \to \frac{\pi}{2}$

$\arctan(0) \to 0$

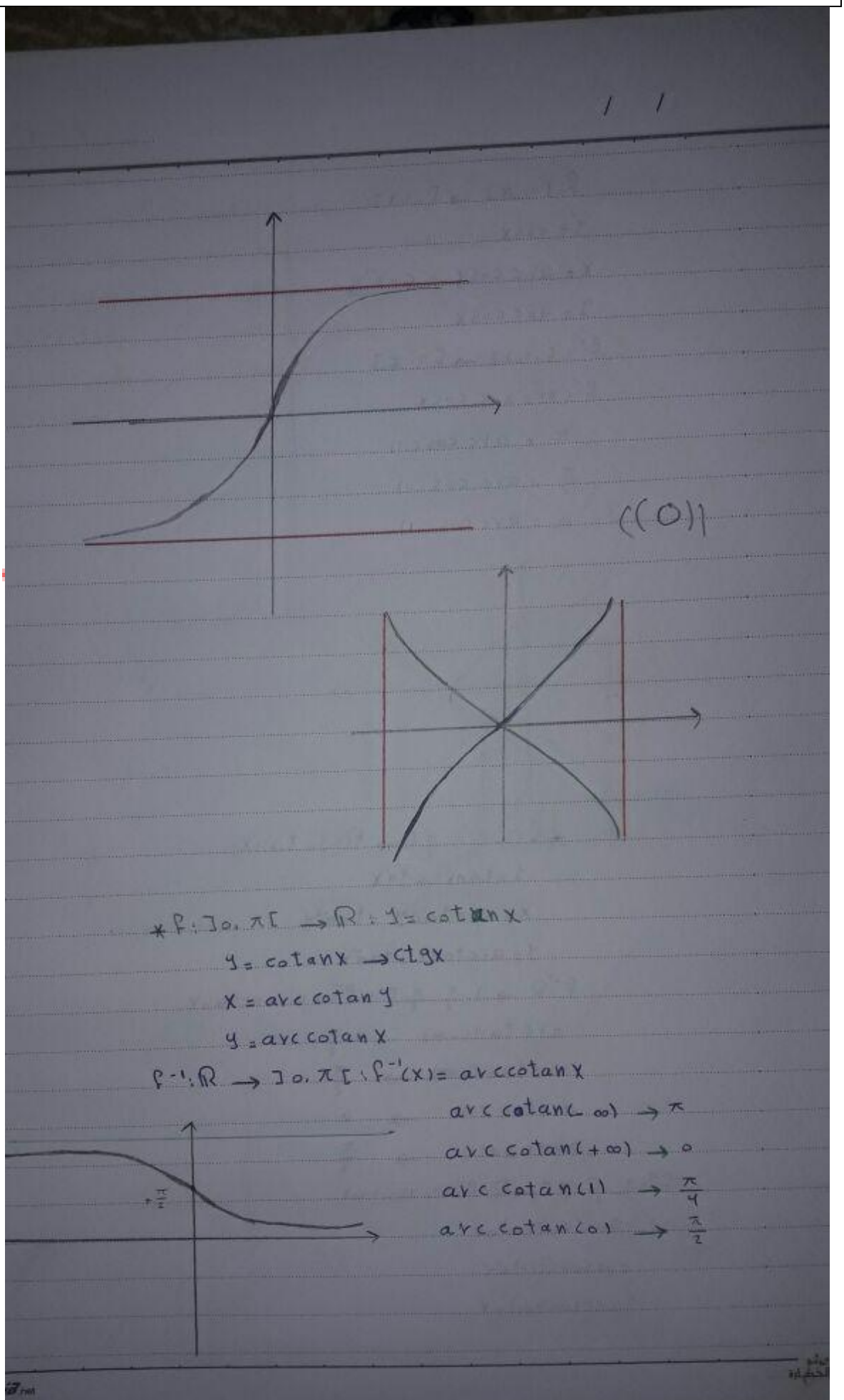
$\arctan(1) \to \frac{\pi}{4}$

$f^{-1}:]-\frac{\pi}{2}, \frac{\pi}{2}[\to \mathbb{R}: f^{-1}(x) = \arctan x$

$\sin(\sin^{-1}(x)) = x$

$\cos(\cos^{-1}(x)) = x$

$\tan(\tan^{-1}(x)) = x$


((0))
* f :]0, π[→ ℝ : y = cotanx
y = cotanx → ctgx
x = arc cotan y
y = arc cotan x
f⁻¹ : ℝ →]0, π[: f⁻¹(x) = arccotanx
arc cotan(-∞) → π
arc cotan(+∞) → 0
arc cotan(1) → π/4
arc cotan(0) → π/2

Syria Math

* مشتقات التوابع العكسية:

$y = \arcsin x \Rightarrow y' = \frac{1}{\sqrt{1-x^2}}$

$y = \arccos x \Rightarrow y' = \frac{1}{\sqrt{1-x^2}}$

$y = \arctan x \Rightarrow y' = \frac{1}{1+x^2}$

$y = \operatorname{arccotan} x \Rightarrow y' = \frac{1}{1+x^2}$

* $f : [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$

$y = \arcsin x$

$x = \sin y$

$1 = \cos y \; y'$

$y' = \frac{1}{\cos y}$

$\cos^2 y + \sin^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \sqrt{1 - \sin^2 y}$

$= \sqrt{1-x^2}$ $\Rightarrow \boxed{y' = \frac{1}{\sqrt{1-x^2}}}$

$y = \arctan x$

$x = \tan y$

$1 = (1 + \tan^2 y)\, y'$

$1 = (1+x^2) y' \Rightarrow \boxed{y' = \frac{1}{1+x^2}}$

$y = \operatorname{arccotan} x$

$x = \operatorname{cotan} y$

$1 = -(1 + \operatorname{cotan}^2 y)\, y'$

$1 = -(1+x^2) y' \Rightarrow \boxed{y' = \frac{-1}{1+x^2}}$

أمثلة:

أحسب ما يلي:

$\cos^{-1}(0)$, $\sin^{-1}(\frac{1}{2})$, $\sec^{-1}(\frac{1}{3})$

$\csc^{-1}(2)$, $\sin^{-1}(-3)$, $\cos(\tan^{-1}(\frac{1}{2}))$

$\cos^{-1}(2)$, $\sec^{-1}(2)$, $\tan^{-1}(\frac{1}{\sqrt{3}})$

$\log_2(32)$, $\tan(\sin^{-1}(x))$

* $\sec^{-1}(\frac{1}{3}) = \text{arc}\sec(\frac{1}{3}) = y$

$\frac{1}{3} = \sec y = \frac{1}{\cos y}$

$\cos y = 3$ مستحيل

* $\cos^{-1}(0)$ $\cos y = 0$

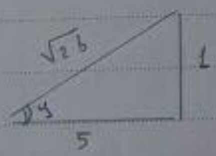
$y = \frac{\pi}{2}$

* $\sin^{-1}(\frac{1}{2}) = y$

$\Rightarrow \frac{1}{2} = \sin y \Rightarrow y = \frac{\pi}{6}$

* $\cos(\tan^{-1}(\frac{1}{5}))$

$\tan^{-1}(\frac{1}{5}) = y \Rightarrow \frac{1}{5} = \tan y$



$\cos y = \frac{5}{\sqrt{26}}$

* $\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$

$\sin^{-1}x = \theta$

* $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{x}{\sqrt{1-x^2}}$