Syria Math

تحليل ١

الدكتور : نايف طلي

المحاضرة :السادسة عشرة

Web: www.syriamath.net

group: Improve our mathematics

/ /

التاريخ: 16

الدوال القطعية

e^{x} , e^{-x}

((16))

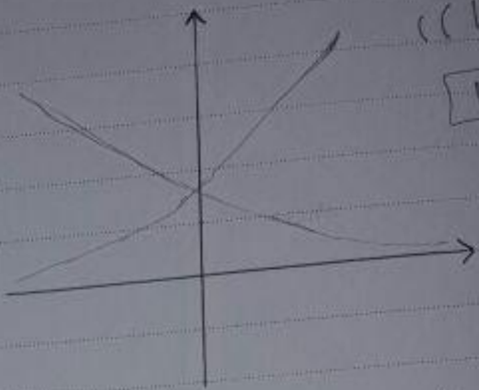
$\boxed{1}$

$sh\,x = \frac{e^{x}-e^{-x}}{2}$

$ch\,x = \frac{e^{x}+e^{-x}}{2}$

$th\,x = \frac{sh\,x}{ch\,x} = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$

$cth\,x = \frac{sh\,x}{ch\,x} = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$



$sh(x)$ فردي ، $ch(x)$ زوجي.

$sh(-x) = \frac{e^{-x}-e^{x}}{2}$ التابع فردي

$= \frac{(e^{x}-e^{-x})}{2}$

$= -\frac{(e^{x}-e^{x})}{2}$

$* \; sh:\mathbb{R} \to \mathbb{R} \; , y = sh\,x$

$arg\,sh:\mathbb{R} \to \mathbb{R} \; , y = arg\,sh\,x$

$\Rightarrow y = \ln(x+\sqrt{x^2+1})$

لنا:

$2y = e^{x} - e^{-x}$

$2y\,e^{x} = e^{2x} - 1$

$e^{2x} - 2y\,e^{x} - 1 = 0$

$A^2 - 2yA - 1 = 0$

$\Delta = 4y^2 - 4(1)(-1) =$

$e^{x} = \frac{2y+2\sqrt{y^2+1}}{2}$

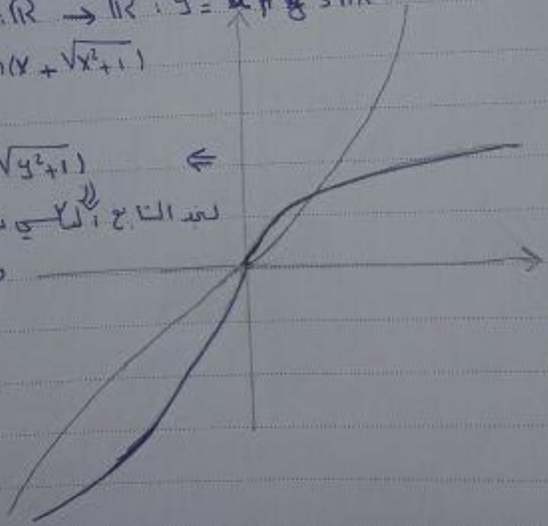
$= y + \sqrt{y^2+1}$

$\neq y - \sqrt{y^2+1}$

$x = \ln(y+\sqrt{y^2+1}) \Leftarrow$

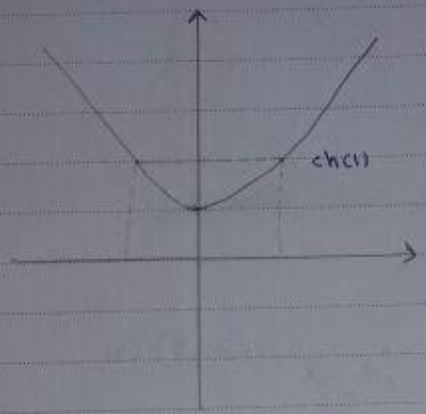
لنعد التابع الأصلي نبدل كل x بـ y

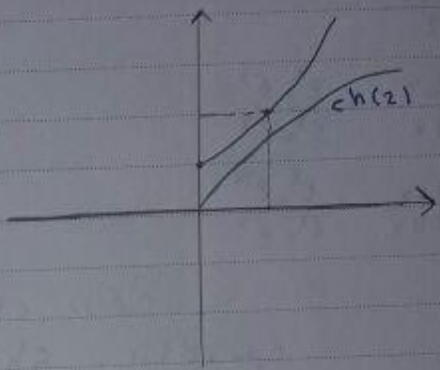
وكل y بـ x



$ch: \mathbb{R} \to [1, +\infty[$

$ch_2: [0, +\infty[\to [1, +\infty[\ : y = ch\,x$





$* \ ch: [0, +\infty[\to [1, +\infty[: y = ch\,x$

$y = ch\,x$

$\boxed{2}$

[illegible]

$2y = e^x + e^{-x}$

$2y e^x = e^{2x} + 1$

$e^{2x} - 2e^x + 1 = 0$

$A^2 - 2yA + 1 = 0$

$\Delta = 4y^2 - 4(1)(1)$

$= 4(y^2 - 1)$

$A = \dfrac{2y + 2\sqrt{y^2-1}}{2}$

$e^x = y + \sqrt{y^2-1}$

$= y - \sqrt{y^2-1}$

$argch: [1, +\infty[\to [0, +\infty[$

$y = argch\,x = \ln(x + \sqrt{x^2-1})$

$* \; th : \mathbb{R} \to]-1, +1[$

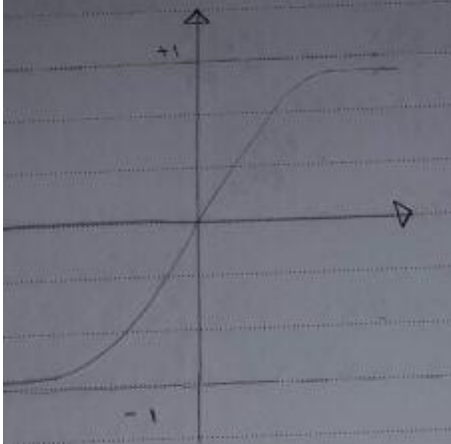
$x \to +\infty \Rightarrow y \to +1$

$x \to -\infty \Rightarrow y \to -1$

ملاحظة:

$$th\, x = \frac{sh\, x}{ch\, x} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$= \frac{e^{2x} - 1}{e^{2x} + 1}$$



$\boxed{3}$

$* \; th : \mathbb{R} \to]-1, +1[$

$arg\,th :]-1, +1[\to \mathbb{R}$

$y = arg\,th\, x$

$y = \frac{1}{2} \ln \frac{1+y}{1-y}$

ملاحظة:

$y(e^{2x} + 1) = e^{2x} - 1$

$y e^{2x} - e^{2x} + y + 1 = 0$

$e^{2x}(y - 1) = -(y + 1)$

$e^{2x}(1 - y) = (1 + y)$

$e^{2x} = \frac{1+y}{1-y}$

$\Rightarrow x = \frac{1}{2} \ln \frac{1+y}{1-y}$

$$coth\, x = \frac{ch\, x}{sh\, x} = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

$* \, cth : \mathbb{R}^* \to \mathbb{R} \setminus [-1,1]$ أي $]-\infty,-1[\cup]1,+\infty[$

$x \to -\infty \Rightarrow y = -1$

ملاحظة:

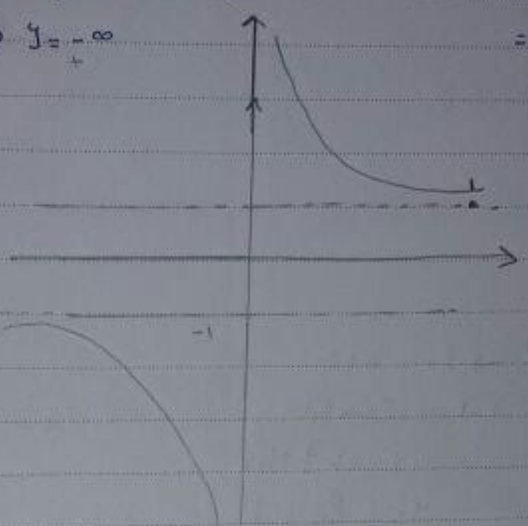
$cth\,x = \dfrac{ch\,x}{sh\,x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

$= \dfrac{e^{2x}+1}{e^{2x}-1}$

$x \to -\infty \Rightarrow y = -1$

$x \to +\infty \Rightarrow y = 1$

$x \to 0 \Rightarrow y = \mp\infty$



4

$* \, argcth : \mathbb{R} \setminus [-1,1] \to \mathbb{R}^*$

$y = argcth\,x$

$y = argcth\,x = \frac{1}{2} \ln \frac{x+1}{x-1}$

ملاحظة:

$y = \dfrac{e^{2x}+1}{e^{2x}-1}$

$\Rightarrow y(e^{2x}-1) = (e^{2x}+1)$

$y(e^{2x}) - y - e^{2x} - 1 = 0$

$e^{2x}(y-1) = y+1$

$e^{2x} = \dfrac{y+1}{y-1}$

$2x = \ln \dfrac{y+1}{y-1}$

$x = \frac{1}{2} \ln \dfrac{y+1}{y-1}$

نبدل x و y نحصل على التابع

Syria Math

$$sh\,x = \frac{e^x - e^{-x}}{2}$$

$$sh\,x = \frac{e^x + e^{-x}}{2}$$

$$th\,x = \frac{sh\,x}{ch\,x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

* $sh\,x + ch\,x = e^x \iff \cos^2 x + \sin^2 x = 1$
* $ch^2 x - sh^2 x = 1$

$\Rightarrow \frac{1}{4}(e^{2x} + 2e^x.e^{-x} + e^{-2x}) - (e^{2x} - 2 + e^{-2x})$

$= \frac{1}{4}[(e^{2x} + 2 + e^{-2x}) - (e^{2x} - 2 + e^{-2x})]$

$= \frac{1}{4}(4) = 1$

$\boxed{5}$

* $ch^2 x + sh^2 x = ch\,2x \iff \cos^2 x + \sin^2 x = 3\cos 2x$

* $ch\,2x = \frac{e^{2x} + e^{-2x}}{2}$

$\frac{1}{4}[(e^{2x} + 2 + e^{-2x}) - (e^{2x} - 2 + e^{-2x})]$

$\frac{1}{4}[2(e^{2x} + e^{-2x})] = \frac{1}{2}(e^{2x} + e^{-2x}) = ch\,2x$

* $2\,sh\,x.\,ch\,x = sh\,2x \iff \sin 2x = 2\sin x.\cos x$

$sh\,2x = \frac{e^{2x} - e^{-2x}}{2}$

$= 2.\left(\frac{e^x - e^{-x}}{2}\right).\left(\frac{e^x + e^{-x}}{2}\right)$

$= \frac{1}{2}(e^{2x} + 1 - 1 - e^{-2x})$

$= \frac{1}{2}(e^{2x} - e^{-2x}) = sh\,2x$