حل التمرين الوارد في المحاضرة السابقة:

$$\frac{X-X_0}{x'} = \frac{y-y_0}{y'} = \frac{3-3_0}{3'}$$

ملحوظة:
إذا كان لدينا قيمة وجب أن نعوض $x_0 = 0$ بـ
$y_0 =$
$3_0 =$
للبرهنة نوعض الدالة ضابع الاشتقاق ثم نعوض الى قيمة t صفر فقط

$$x_0 = 0 \Leftarrow$$
$$y_0 = 0$$
$$3_0 = a$$

$$x' = -a\sin^2 t + a\cos^2 t$$
$$x'_0 = a$$
$$y' = 2a\cos t \cdot \sin t$$
$$y'_0 = 0$$
$$3' = -a\sin t$$
$$3'_0 = 0$$

$$x'' = -2a\sin t\cos t - 2a\sin t\cos t$$
$$x'' = -4a\sin t\cos t$$
$$x''_0 = 0$$

$$y'' = -2a\sin^2 t + 2a\cos^2 t$$
$$3'' = -a\cos t$$
$$3''_0 = -a$$

$$x = 0$$
$$\frac{y}{2a} = \frac{3-a}{-a}$$

$$x'(x-x_0) + y'(y-y_0) + 3'(3-3_0) = 0$$

في هذه الحالة المحور الجانبي $x = 0$

نجد $2ay - a(3-a) = 0$

$$\begin{vmatrix} x-x_0 & y-y_0 & 3-3_0 \\ x' & y' & 3' \\ x'' & y'' & 3'' \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y & 3-a \\ a & 0 & 0 \\ 0 & 2a & -a \end{vmatrix} = 0$$

Scanned by CamScanner

حل التمرين الثاني

$$x' = 2t \Rightarrow x'' = 2$$
$$y' = 3t^2 \Rightarrow y'' = 6T$$
$$3' = \frac{36}{16}T^3 \Rightarrow 3'' = \frac{108}{16}T^2$$

$$\vec{T} = \begin{cases} \alpha_1 = \frac{dx}{ds} \\ \beta = \frac{dy}{ds} \\ \gamma = \frac{dz}{ds} \end{cases}$$

$$ds = \sqrt{(dx)^2 + (dy)^2}$$ يكن

$$ds = \sqrt{(2T)^2 + (3T^2)^2 + \left(\frac{36}{16}T^3\right)^2}$$

$$ds = a$$

$$\vec{T} = \left(\frac{2T}{a}, \frac{3T^2}{a}, \frac{\frac{36}{16}T^3}{a}\right)$$

$$\vec{N} = \frac{d\vec{T}}{da} = \begin{cases} \alpha_1 = \frac{d\alpha}{a} \\ \beta_1 = \frac{d\beta}{a} \\ \gamma_1 = \frac{d\gamma}{a} \end{cases}$$

$$da = \sqrt{(d\alpha)^2 + (d\beta)^2 + (d\gamma)^2}$$

$$= \sqrt{\frac{2}{a^2} + \left(\frac{6T}{a}\right)^2 + \left(\frac{108T^2}{16a}\right)^2}$$

$$da = b$$

Scanned by CamScanner

$$\vec{N} = \left(\frac{2}{ab}, \frac{6T}{ab}, \frac{108T^2}{16ab} \right)$$

$$\vec{B} = \vec{T} \wedge \vec{N}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & B & \gamma \\ x_1 & B_1 & \gamma_1 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{2t}{a} & \frac{3t^2}{a} & \frac{36T}{16a} \\ \frac{2}{ab} & \frac{6T}{ab} & \frac{108T}{16ab} \end{vmatrix}$$

$$\rho = \frac{(x'^2 + y'^2 + 3'^2)^{\frac{3}{2}}}{(A^2 + B^2 + C^2)^{\frac{1}{2}}}$$

x ← $A = y'3'' - 3'y''$

y ← $B = 3'x'' - x'3''$

3 ← $C = x'y'' - x''y'$

$$x'_{t=1} = 2, \quad y'_{t=1} = 3 \quad 3'_{t=1} = \frac{36}{16}$$

$$x''_{t=1} = 2 \quad y''_{t=1} = 6 \quad 3''_{t=1} = \frac{108}{16}$$

التمرين الثالث :

$$x' = e^t \sin t + e^t \cos t$$
$$y' = e^t \cos t + e^t \sin t$$
$$3' = e^t$$

$$S = \int_0^t \sqrt{(x'^2 + y'^2 + 3'^2)}\, dt$$

$$x'^2 = e^{2t} \sin^2 t + e^{2t} \cos 2t - 2e^t \sin t$$
$$= e^{2t} [\sin^2 t - 2 \sin t + \cos t^2]$$
$$= e^{2t} [1 - 2 \sin t \cos t]$$

Scanned by CamScanner

$$y'^2 = e^{2t}[1 + 2\sin t \cos t]$$

$$3'^2 = e^{2t}$$

$$x'^2 + y'^2 + 3'^2 = e^{2t}[1 + 1 + 1] = 3e^{2t}$$

$$S = \int_0^t \sqrt{3e^{2t}}\, dt = \int_0^t \sqrt{3}e^t\, dt$$

$$= \sqrt{3e^t} - \sqrt{3} = S$$

$$S = \sqrt{3}(e^t - 1)$$

$$\frac{S}{\sqrt{3}} = e^t - 1$$

$$\frac{S+\sqrt{3}}{\sqrt{3}} = e^t \Rightarrow t = \ln\left(\frac{S+\sqrt{3}}{\sqrt{3}}\right)$$

بفوض بن حصاد الله الشفاء .

التمرين الرابع :

$$x = 2t$$
$$y = t^2$$
$$3 = \ln t$$

$$A \in C \Leftarrow \begin{cases} 2 = 2t \\ 1 = t^2 \\ 0 = \ln t \end{cases} \Rightarrow \boxed{t = 1}$$

$$B \in C \Leftarrow \begin{cases} 4 = 2t \\ 4 = t^2 \\ \ln 2 = \ln t \end{cases} \Rightarrow \boxed{t = 2}$$

$$S = \int_{t=1}^{t=2} \sqrt{x'^2 + y'^2 + 3'^2}\, dt$$

اعداد: عبدالله عطفي
فريق سيرياسات

$$S = \int^2 \sqrt{4 + 4t^2 + \frac{1}{t^2}}\, dt$$

Scanned by CamScanner