المحاضرة الثامنة

الأحد 4/12/2017 سمير ... [illegible]

الدوال المثلثية:

$$\int f(\sin x \cdot \cos x)\,dx$$

$$t = \tan\frac{x}{2}$$

$$\frac{x}{2} = \arctan t$$

$$\Rightarrow x = 2\arctan t$$

$$dx = \frac{2dt}{1+t^2}$$

$$\sin x = 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}$$

$$= \frac{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}}$$

$$= \frac{\frac{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}{\cos^2\frac{x}{2}}}{\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}}{\cos^2\frac{x}{2}}}$$

$$\sin x = \frac{2\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{\frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}+1}$$

$$\sin x = \frac{2t}{t^2+1}$$

$$\cos x = \frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}$$

نقسم البسط والمقام على $\cos^2\frac{x}{2}$

$$= \frac{\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}}{\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}}$$

$$\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\boxed{\cos x = \frac{1-t^2}{1+t^2}}$$

[illegible]

التحويل العام لإجراء التكاملات المثلثية

$$t = \tan\frac{x}{2} \Rightarrow x = 2\arctan t$$

$$dx = \frac{2dt}{1+t^2}$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

أوجد تكامل

$$I = \int \frac{dx}{\sin x}$$

$$t = \tan\frac{x}{2}$$

$$I = \int \frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}}$$

$$= \int \frac{dt}{t} = \ln|t| + c$$

$$= \ln\left|\tan\frac{x}{2}\right| + c$$

$$I = \int \frac{dx}{\cos x}$$

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$$I = \int (\tan \tfrac{x}{2})^3 \, dx$$

$$t = \tan \frac{x}{2}$$

$$I = \int t^3 \cdot \frac{2\,dt}{1+t^2}$$

$$I = 2\int \frac{t^3\,dt}{1+t^2}$$

$$\begin{array}{r|l} t^3 & 1+t^2 \\ \hline t^3+t & t \\ \hline -t & \end{array}$$

$$= 2\int \left( t + \frac{-t}{1+t^2} \right) dt$$

$$= 2\left( \frac{t^2}{2} - \frac{1}{2} \ln|1+t^2| + c \right)$$

$$= \frac{\tan^2 x}{2} - \ln|1+\tan^2 \tfrac{x}{2}| + c$$

الدالة الثالثة

$$I = \int \cot x^3 \, dx$$

$$\cot x = t$$

$$x = \text{arc}\cot t$$

$$dx = \frac{-dt}{1+t^2}$$

$$I = -\int t^3 \frac{dt}{1+t^2}$$

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$$I = \int \cos^4 x \cdot \sin^3 x \, dx$$

$$= \int \cos^4 x \, \sin^2 x \cdot \sin x \, dx$$

$$= \int \cos^4 x \, (1-\cos^2 x) \cdot \sin x \, dx$$

$$= \cos x = t$$

$$-\sin x \, dx = dt$$

$$I = -\int t^4 (1-t^2)\, dt$$

$$= -\int (t^4 - t^6)\, dt$$

$$= -\frac{t^5}{5} + \frac{t^7}{7} + c$$

$$= -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + c$$

$$= 2\int \frac{(2t - 1 + t^2)}{(1+t+1-t^2)(1+t^2)}\, dt$$

$$= 2\int \frac{t^2 - 2t - 1}{1 - t^2}\, dt$$

$$= \int \left(1 + \frac{2t - 2}{t^2 + 1}\right) dt$$

$$= \int 1 + \frac{2t}{1+t^2} - 2\,\frac{1}{1+t^2}\, dt$$

$$= t + \ln|1+t^2| - 2\arctan t + C$$

$$= \tan\frac{x}{2} + \ln\left|1 + \tan^2\frac{x}{2}\right| - 2\arctan\left(\tan\frac{x}{2}\right) + C$$

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$$I = \int \frac{\sin^4 x}{\cos^4 x}\, dx$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$I = \int \sin^4 x\, dx$$

$$= \int \left(\frac{1 - \cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int (1 - 2\cos 2x + \cos^2 2x)\, dx$$

$$= \frac{1}{4}\int \left(1 - 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x\right) dx$$

$$= \frac{1}{4}\left(x - \sin 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x\right) + C$$

$$= \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

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$$I = \int \frac{\sin x - \cos x}{1 + \cos x}\, dx$$

$$t = \tan\frac{x}{2}$$

$$I = \int \frac{\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}}{1 + \frac{1-t^2}{1+t^2}} \cdot \frac{2\,dt}{1+t^2}$$

$$I=\int \frac{dx}{1+\sin x+\cos x}$$

$$t=\tan\frac{x}{2}$$

$$=\int \frac{\frac{2dt}{1+t^2}}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}$$

$$=2\int \frac{dt}{1+t^2+2t+1-t^2}$$

$$=2\int \frac{dt}{2+2t}$$

$$=\ln|2+2t|+c$$

$$=\ln\left|2+2\tan\frac{x}{2}\right|+c$$

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$$I=\int \frac{dx}{2\sin x-\cos x+5}$$

$$t=\tan\frac{x}{2}$$

$$I=\int \frac{\frac{2dt}{1+t^2}}{2\,\frac{2t}{1+t^2}-\frac{1-t^2}{1+t^2}+5}$$

$$=\int \frac{2dt}{6t^2+4t+4}$$

$$=\int \frac{dt}{3t^2+2t+2}$$

$$I=\int \frac{\sin x}{\sin x+\cos x}\,dx$$

$$t=\tan\frac{x}{2}$$

$$I=\int \frac{\frac{2t}{1+t^2}\cdot\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}$$

$$=4\int \frac{t\,dt}{(1+t^2)(-t^2+2t+1)}$$

$$=4\int \frac{t\,dt}{(1+t^2)(t+1+\sqrt{2})(t+1-\sqrt{2})}$$

$$=\frac{At+B}{1+t^2}+\frac{C}{t+1+\sqrt{2}}+\frac{D}{t+1-\sqrt{2}}$$

$$= \frac{1}{3}\int \frac{dt}{\left(t^2 + \frac{2}{3}t + \frac{1}{9}\right) + \frac{5}{9}}$$

$$= \frac{1}{3}\int \frac{dt}{\left(t + \frac{1}{3}\right)^2 + \frac{5}{9}}$$

$$= \frac{13}{3\sqrt{5}} \arctan \frac{t + \frac{1}{3}}{\sqrt{\frac{5}{9}}}$$

إعداد الطالبة

رشا رجب