2017/10/24 ن   المحاضرة السابعة

$$+\ \psi(x) = \cot g\, x + \lambda \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan g\, t \ \psi(t)\, dt$$

$\frac{36}{142}$

$k(x,t) = \tan g\, t$

$a_1(x) = 1 \qquad b_1(t) = \tan g(t)$

$$\int_a^b \int_a^b |k(x,t)|^2\, dx\, dt = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |\tan g(t)|^2\, dx\, dt$$

$$= \pi - \frac{\pi^2}{4} < \infty$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |k(x,t)|^2\, dx = \frac{\pi}{2}\, tg^2(t) < \infty$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |k(x,t)|^2\, dt = 2 - \frac{\pi}{2} < \infty$$

$$a_{11} = \int_a^b b_1(t)\, a_1(t)\, dt = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan(t)\, dt = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin t}{\cos t}\, dt$$

$$= \left[-\ln(\cos t)\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = 0$$

$$h_1 = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} b_1(t)\, h(t)\, dt = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan(t)\cot(t)\, dt =$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1\, dt = \frac{\pi}{2}$$

$$\Rightarrow (1 - \lambda a_{11})\, c_1 = h_1 \Rightarrow c_1 = h_1 = \frac{\pi}{2}$$

$$\Rightarrow \varphi(x) = \cot g(t) + \lambda \frac{\pi}{2}$$

* $\varphi(x) = \lambda \int_{-1}^{1} |x|\, \varphi(t)\, dt$

$h(x) = 0 \qquad k(x,t) = |x| \qquad a_1(x) = |x| \qquad b_1(t) = 1$

$$\int_{-1}^{1}\int_{-1}^{1} (|x|^2)\, dx\, dt = \int_{-1}^{1} \left[\frac{x^3}{3}\right]_{-1}^{1} dt = \frac{2}{3}\int_{-1}^{1} dt$$

$$= \frac{2}{3}[t]_{-1}^{1} = \frac{4}{3} < \infty$$

$$\int_{-1}^{1} (|x|^2)\, dx = \left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{2}{3} < \infty$$

$$\int_{-1}^{1} (|x|^2)\, dt = |x|^2 [t]_{-1}^{1} = 2x^2 < \infty$$

$$a_{11} = \int_{-1}^{1} b_1(t)\, a(t) = \int_{-1}^{1} |t|\, dt = \int_{-1}^{0} -t\, dt + \int_0^1 t\, dt$$

$$= \left[-\frac{t^2}{2}\right]_{-1}^{0} + \left[\frac{t^2}{2}\right]_0^1 = \frac{1}{2} + \frac{1}{2} = 1$$

$$h = \int_{-1}^{1} b_1(t)\, h(t)\, dt = 0$$

$$(1 - \lambda a_{11})\, c_1 = h_1 \Rightarrow (1 - \lambda)\, c_1 = 0 \Rightarrow c_1 = 0$$

$$\Rightarrow \varphi(x) = h(x) + \lambda c_1 a_{11} = 0 + 0 = 0 \Rightarrow \varphi(x) = 0$$

نستنتج أن المعادلة المتجانسة لها الحل الصفري .

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$$\psi(x) = \lambda_0 \int_0^1 \sin(\ln x)\, \psi(t)\, dt = 2x$$

$k(x,t) = \sin(\ln x)$ $\quad a_1(x) = \sin(\ln x)$ $\quad b_1(t) = 1$

$$\int_0^1 \int_0^1 \sin^2(\ln x)\, dx\, dt < \infty$$

$$a_{11} = \int_0^1 b_1(t)\, a_1(t) = \int_0^1 \sin(\ln t)\, dt = -\frac{1}{2}$$

+ وظيفة إيجاد $\int_0^1 \sin(\ln t)\, dt$ (تكامل بعموم التكاملات المعتلة تحليل 3)

$$h_1 = \int_0^1 b_1(t)\, h(t)\, dt = \int_0^1 2t\, dt = 1$$

$$(1 - \lambda a_{11})\, c_1 = h_1 \Rightarrow 1 - \lambda\left(\frac{-1}{2}\right) c_1 = 1$$

$$\Rightarrow 1 + \lambda \frac{c_1}{2} = 1$$

$$\Rightarrow \left(\frac{2+\lambda}{2}\right) c_1 = 1$$

$$\Rightarrow c_1 = \frac{2}{2+\lambda}$$

$$\Rightarrow \psi(x) = h(x) + c_1 \lambda\, a_1(x)$$

$$= 2x + \frac{2\lambda}{2+\lambda} \sin(\ln(x))$$

+ حل معادلة فريدهولم التكاملية المتجانسة بطريقة النواة المتردية :

هو نفس حل معادلة فريدهولم التكاملية الغير المتجانسة لكن بوضع $h = 0$

حيث المعادلة المتجانسة تكتب بالشكل $\psi(x) = \lambda \int_a^b k(x,t)\, \psi(t)\, dt$

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SABBAGH STATIONERY

$$\psi(x) - \lambda \int_0^1 (3x-2)\, t\, \psi(t)\, dt = 0$$

$a_1(x) = 3x \qquad b_1(t) = t$

$a_2(x) = 1 \qquad b_2(t) = -2t$

$$a_{11} = \int_0^1 3.t.t\, dt = 3\int_0^1 t^2 dt = 1$$

$$a_{21} = \int_0^1 (-2t)(3t)\, dt = -6\int_0^1 t^2 dt = -2$$

$$a_{12} = \int_0^1 t\, dt = \frac{1}{2}$$

$$a_{22} = -2\int_0^1 t\, dt = -1$$

$$(1-\lambda)c_1 - \frac{\lambda}{2} c_2 = 0$$

$$2\lambda\, c_1 + (1+\lambda)\, c_2 = 0$$

$$\Delta(\lambda) = \begin{vmatrix} 1-\lambda & -\frac{\lambda}{2} \\ 2\lambda & 1+\lambda \end{vmatrix} = 1 \neq 0$$

$\psi(x) = 0$ إذاً حلها الوحيد هو الحل الصفري

ولا توجد أي قيمة مميزة لـ λ