المحاضرة الثالثة

اثنين 9/10/2017

$-\left(\frac{2}{90}\right)$

$$\varphi(x) = \frac{1}{2}\int_0^1 \varphi(t)\,dt + \sin \pi x$$

$$h(x) = \sin \pi x$$

$$a = 0 \qquad b = 1 \qquad k(x,t) = 1$$

$$\int_a^b \int_a^b |k(x,t)|^2\,dx\,dt < \infty$$

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$$\int_a^b |h(x)|^2\,dx < \infty$$

$$\int_0^1\int_0^1 dx\,dt = \int_0^1 [x]_0^1\,dt = \int_0^1 dt = [t]_0^1 = 1$$

$$\int_0^1 1^2\,dx = [x]_0^1 = 1$$

$$\int_0^1 1^2\,dt = [t]_0^1 = 1$$

$$\int_0^1 |\sin \pi x|^2\,dx = \int_0^1 \frac{1 - \cos 2\pi x}{2}\,dx$$

$$= \left[\frac{1}{2}x\right]_0^1 - \frac{1 \sin 2\pi x}{4\pi}$$

$$= \frac{1}{2} < \infty$$

نستنتج أن جميع الشروط محققة وبالتالي يمكن حل هذه المعادلة التكاملية المعروفة بنظرية النظريات المتتالية.

$$\varphi_0(x) = h(x) = \sin \pi x$$

$$\varphi_1(x) = \sin \pi x + \frac{1}{2}\int_0^1 \varphi_0(t)\, dt$$

$$= \sin \pi x + \frac{1}{2}\int_0^1 \sin \pi t \, dt$$

$$= \sin \pi x + \frac{1}{2}\left[-\frac{1}{\pi} \cos \pi t \right]_0^1$$

$$= \sin \pi x + \frac{1}{2}\left[\frac{1}{\pi} + \frac{1}{\pi} \right]$$

$$= \sin \pi x + \frac{1}{\pi}$$

$$\varphi_2(x) = \sin \pi x + \frac{1}{2}\int_0^1 \sin \pi t + \frac{1}{\pi} \, dt$$

$$= \sin \pi x + \frac{1}{2}\left[\frac{-1}{\pi} \cos \pi t + \frac{t}{\pi} \right]_0^1$$

$$= \sin \pi x + \frac{1}{\pi} + \frac{1}{2\pi}$$

$$\varphi_3(x) = \sin \pi x + \frac{1}{2}\int_0^1 \left(\sin \pi t + \frac{1}{\pi} + \frac{1}{2\pi} \right) dt$$

$$= \sin \pi x + \frac{1}{2}\left[\frac{-1}{\pi} \cos \pi t + \frac{t}{\pi} + \frac{t}{2\pi} \right]_0^1$$

$$= \sin \pi x + \frac{1}{2}\left[\frac{2}{\pi} + \frac{1}{\pi} + \frac{1}{2\pi} \right]$$

$$\varphi_3(x) = \sin \pi x + \frac{1}{\pi} + \frac{1}{2\pi} + \frac{1}{2^2 \pi}$$

$$\varphi_n(x) = \sin \pi x + \sum_{i=1}^{n} \frac{1}{2^{i-1} \pi}$$

$$\lim \varphi_n(x) = \varphi(x) = \sin \pi x + \sum_{i=1}^{\infty} \frac{1}{2^{i-1}\pi}$$

$$S = \frac{1}{1-\frac{1}{2}} = 2 \Rightarrow \varphi(x) = \sin \pi x + \frac{2}{\pi}$$

هل هذه المعادلة قابلة للحل وفي حال الإيجاب أوجد باستخدام نيومن ؟

$$B^2 = \int_0^1 \int_0^1 k(x,t)\, dx\, dt$$

$$B = 1$$

$$\lambda = \frac{1}{2} < \frac{1}{B} = 1$$

⇐ قابلة للحل.

$$k_1(x,t) = k(x,t) = 1$$

$$k_2(x,t) = \int_0^1 k_1(x,z)\, k(z,t)\, dz = 1$$

$$\vdots$$

$$k_n(x,t) = 1$$

$$\varphi(x) = h(x) + \sum \lambda^m \int_a^b k_m(x,t)\, h(t)\, dt$$

شرط نيومن (29) ص 30

$$\varphi(x) = \sin \pi x + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \int_0^1 1 \sin \pi t\, dt$$

$$= \sin \pi x + \sum \left(\frac{1}{2}\right)^n \left[\frac{-1}{\pi} \cos \pi t\right]_0^1$$

$$= \sin \pi x + \left(\frac{1}{\pi} + \frac{1}{\pi}\right)$$

$$= \sin \pi x + \frac{2}{\pi}$$

مجموع متسلسلة $\frac{\frac{1}{2}}{1-\frac{1}{2}}$

$$* \quad \psi(x) = 1 + \lambda \int_0^1 x.t \; \psi(t) \, dt$$

أوجد $k_m(x,t)$ النوى المكررة

$$k_1(x,t) = k(x,t) = x.t$$

$$k_2(x,t) = \int_0^1 k_1(x,z) \, k_1(z,t) \, dz$$

$$= \int_0^1 x.z \; . \; z.t \; dz$$

$$= \int_0^1 x.z^2.t \; dz$$

$$= x.t \int_0^1 z^2 \, dz$$

$$= x.t \left[\frac{z^3}{3}\right]_0^1$$

$$= x.t \; \frac{1}{3}$$

$$k_3(x,t) = \int_0^1 k_2(x,z) \; k_1(z,t) \, dz$$

$$= \int_0^1 \frac{x.z}{3} \; . \; z.t \; dz$$

$$= \left[\frac{x.t}{9} z^3\right]_0^1 = \frac{x.t}{9} = \frac{x.t}{3^2}$$

$$k_4(x,t) = \int_0^1 k_3(x,z) \, k(z,t) \, dz$$

$$= \int_0^1 \left(\frac{x.z}{9}\right) (z.t) \; dz$$

$$= \frac{x.t}{3^3} = \frac{1}{3^3} k(x,t)$$

$$k_n(x,t) = \frac{x.t}{3^{m-1}} \quad \cdots \quad = \frac{1}{3^{m-1}} k(x,t)$$

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تأكد $\int \cos z \sin z \, dz = 0$

$*\quad \varphi(x) = \cos 2x + \int_0^{2\pi} \sin x \cos t \, \varphi(t) \, dt$

أوجد $k_m(x,t)$

$k_1(x,t) = k(x,t) = \sin x \cos t$

$k_2(x,t) = \int_0^{2\pi} k_1(x,z) \, k_1(z,t) \, dz$

$= \int_0^{2\pi} \sin x \cos z \sin z \cos t \, dz$

$= \sin x \cos t \int_0^{2\pi} \underbrace{\cos z \sin z \, dz}_{0}$

$k_3(x,t) = 0$

$\vdots$

$*\quad \varphi(x) = 1 + \lambda \int_0^1 (1 - 3xt) \, \varphi(t) \, dt$ ص 127

باستخدام التقريبات المتتالية.

$\int_0^1 \int_0^1 (1 - 3xt)^2 \, dx \, dt$

$= \int_0^1 \int_0^1 1 - 6xt + 9x^2t^2 \, dx \, dt$

$\int_0^1 \left[x - \frac{6x^2}{2} t + 9 \frac{x^3}{3} t^2 \right]_0^1 dt$

$\int_0^1 [1 - 3t + 3t^2] \, dt$

$\left[t - \frac{3}{2} t^2 + \frac{3}{3} t^3 \right]_0^1$

$= \left[1 - \frac{3}{2} + 1 \right]$

$2 - \frac{3}{2} = \frac{4}{2} - \frac{3}{2} = \frac{1}{2}$

حل المعادلة التكاملية:

$$\psi(x) = 1 + \lambda\int_0^1 (1 - 3xt)\psi(t)dt$$

الحل:

١ . طريقة التقريبات المتتالية:

لا بد من حساب B^2.

$$B^2 = \int_a^b\int_a^b |k(x,t)|^2 dxdt$$

$$B^2 = \int_0^1\int_0^1 |1 - 3xt|^2 dxdt = \int_0^1 \left[x + 3x^3t^2 - 3x^2t\right]_0^1 dt$$

$$= \int_0^1 \left[1 + 3t^2 - 3t\right]dt = \left[t + t^3 - 3\frac{t^2}{2}\right]_0^1 = \frac{1}{2}$$

لدينا:

$$\psi(x) = h(x) + \lambda\int_a^b k(x,t)\psi(t)dt$$

نفرض $\psi_0(x) = h(x)$ عندئذ نجد:

$$\psi_1(x) = h(x) + \lambda\int_a^b k(x,t)\psi(t)dt$$

$$= 1 + \lambda\int_0^1 (1 - 3xt)(1)dt = 1 + \left[\lambda\left(\frac{2 - 3x}{2}\right)\right]$$

$$\psi_2(x) = h(x) + \lambda\int_a^b k(x,t)\psi_1(t)dt$$

$$= 1 + \lambda\int_0^1 (1 - 3xt)\left(1 + \lambda\left(\frac{2 - 3t}{2}\right)\right)dt$$

$$=1+\lambda\int_0^1(1-3xt)\left(1+\lambda-\frac{3t\lambda}{2}\right)dt$$

$$=1+\lambda\int_0^1(1+\lambda-\frac{3t\lambda}{2}-3xt-3xt\lambda+9\frac{xt^2\lambda}{2})dt$$

$$\psi_2(x)=1+\lambda\left[t+\lambda t-3\frac{t^2\lambda}{4}-3x\frac{t^2}{2}-\frac{3xt^2\lambda}{2}+\frac{3x\lambda t^3}{2}\right]_0^1$$

$$\psi_2(x)=1+\lambda\left[1+\lambda-3\frac{\lambda}{4}-\frac{3x}{2}-\frac{3x}{2}\lambda+\frac{3x}{2}\lambda\right]$$

$$=1+\lambda+\frac{1}{4}\lambda^2-\frac{3x}{2}\lambda=1+\lambda\left(1-\frac{3x}{2}\right)+\frac{1}{4}\lambda^2$$

$$\psi_3(x)=1+\lambda\int_a^b k(x,t)\psi_2(t)dt$$

$$=1+\lambda\int_0^1(1-3xt)\left[1+\lambda\left(1-\frac{3t}{2}\right)+\frac{1}{4}\lambda^2\right]dt$$

$$=1+\lambda\int_0^1\left[1+\lambda\left(1-\frac{3t}{2}\right)+\frac{1}{4}\lambda^2\right]dt$$

$$-3xt\lambda\int_0^1\left[1+\lambda\left(1-\frac{3t}{2}\right)+\frac{1}{4}\lambda^2\right]dt$$

«فرقنا التكامل لتكاملين».

$$\psi_3(x)=1+\lambda\left(1-\frac{3x}{2}\right)+\frac{1}{4}\lambda^2+\frac{1}{4}\lambda^3\left(1-\frac{3x}{2}\right)$$

$$\psi_4(x)=1+\lambda\int_0^1(1-3xt)\psi_3(t)dt$$

$$\psi_4(x)=1+\lambda\left(1-\frac{3}{2}x\right)+\frac{1}{4}\lambda^2+\frac{\lambda^3}{4}\left(1-\frac{3x}{2}\right)+\left(\frac{\lambda^2}{4}\right)^2$$

$$=1+\left(1-\frac{3x}{2}\right)\left(\lambda+\frac{\lambda^3}{4}\right)+\frac{\lambda^2}{4}\left(1+\frac{\lambda^2}{4}\right)$$

$$\psi_5(x)=1+\lambda\int_0^1(1-3xt)\psi_4(t)dt$$

$$=1+\lambda\left(1-\frac{3x}{2}\right)+\frac{1}{4}\lambda^2+\frac{\lambda^3}{4}\left(1-\frac{3x}{2}\right)+\left(\frac{\lambda^2}{4}\right)^2+\frac{\lambda^5}{16}\left(1-\frac{3x}{2}\right)$$

$$=1+\left(1-\frac{3}{2}x\right)\left[\lambda+\frac{\lambda^3}{4}+\frac{\lambda^5}{16}\right]+\frac{\lambda^2}{4}\left[1+\frac{\lambda^2}{4}\right]$$

ونحسب $\psi_6(x)$ بنفس الأسلوب السابق فنجد:

$$\psi_6=1+\lambda\left(1-\frac{3x}{2}\right)+\frac{1}{4}\lambda^2+\frac{\lambda^3}{4}\left(1-\frac{3x}{2}\right)+\left(\frac{\lambda^2}{4}\right)^2+\frac{\lambda^5}{16}\left(1-\frac{3x}{2}\right)+\frac{\lambda^6}{4\cdot16}$$

$$\psi_6(x)=1+\left(1-\frac{3}{2}x\right)\left[\lambda+\frac{\lambda^3}{4}+\frac{\lambda^5}{16}\right]+\frac{\lambda^2}{4}\left[1+\frac{\lambda^2}{4}+\frac{\lambda^4}{16}\right]$$

«وهكذا حتى $\psi_n(x)$».

الآن نقوم بترتيب الحدود بالشكل التالي:

$$\lim_{n\to\infty}\psi_n(x)=1+\left(1-\frac{3x}{2}\right)\left(\lambda+\frac{\lambda^3}{4}+\frac{\lambda^5}{16}+\frac{\lambda^7}{64}+\ldots\right)$$

$$+\left(\frac{1}{4}\lambda^2+\left(\frac{\lambda^2}{4}\right)^2+\frac{\lambda^6}{4\cdot16}+\ldots\right)$$

$$=1+\lambda\left(1-\frac{3x}{2}\right)\left(1+\frac{\lambda^2}{4}+\frac{\lambda^4}{16}+\frac{\lambda^6}{64}+\ldots.\right)$$

$$+\left(\frac{1}{4}\lambda^2+\left(\frac{\lambda^2}{4}\right)^2+\frac{\lambda^6}{4\cdot16}+\ldots\right)$$

$$=\lambda\left(1-\frac{3x}{2}\right)\left(1+\frac{\lambda^2}{4}+\frac{\lambda^4}{16}+\frac{\lambda^6}{64}+\ldots\right)+$$

$$+\left(1+\frac{\lambda^2}{4}+\frac{\lambda^4}{16}+\frac{\lambda^6}{64}+\ldots\right)$$

$$\psi(x)=\lim_{n\to\infty}\psi_n(x)=1+\lambda\left(1-\frac{3x}{2}\right)\sum_{n=0}^{\infty}\left(\frac{\lambda^2}{4}\right)^n+\frac{\lambda^2}{4}\sum_{n=0}^{\infty}\left(\frac{\lambda^2}{4}\right)^n$$

لكن نلاحظ أن: $\left(1+\frac{\lambda^2}{4}+\frac{\lambda^4}{16}\ldots\right)$ هي متسلسلة هندسية أساسها $\frac{\lambda^2}{4}$

ومنه مجموع المتسلسلة الهندسية يعطى بالعلاقة:

$$\sum_{n=0}^{\infty}\left(\frac{\lambda^2}{4}\right)^n=\frac{1}{1-\frac{\lambda^2}{4}}=\frac{4}{4-\lambda^2}$$

$$\psi|x|=\lim_{n\to\infty}\psi_n(x)=1+\frac{4\lambda}{4-\lambda^2}\left[1-\frac{3}{2}x+\frac{\lambda}{4}\right]=\frac{4+2\lambda(2-3x)}{4-\lambda^2}$$

ومنه الحل يصلح باستثناء القيم التي تعدم المقام.