

المورفيزمات المألوفة

تعريف: لتكن L_1, L_2 فئتين و $F, G: L_1 \rightarrow L_2$ دوال عابرة
 فنقول انه يوجد لدينا مورفيزم $f: F \rightarrow G$ من الدالي F الى الدالي G
 اذا كان $\forall x \in \text{ob}(L_1)$ $f(x) = G(x) \circ F(x)$

نصف: $f(x): F(x) \rightarrow G(x)$ مورفيزم للفئة L_2
 نجد كل مورفيزم $u: A \rightarrow B$ للفئة L_1 الخلفه التي تبدي $A, B \in \text{ob}(L_1)$

$$\begin{array}{ccc} F(A) & \xrightarrow{f(A)} & G(A) \\ F(u) \downarrow & & \downarrow G(u) \\ F(B) & \xrightarrow{f(B)} & G(B) \end{array}$$

لكل تبدي u أي $G(u) \circ f(A) = f(B) \circ F(u)$

تعريف

للمر $F, G: L_1 \rightarrow L_2$ دوال غير مباشرة

نقول انه يوجد لدينا مورفيزم $f: F \rightarrow G$ اذا كان $\forall x \in \text{ob}(L_1)$

$\forall x \in \text{ob}(L_1)$ نصف $f(x): F(x) \rightarrow G(x)$ مورفيزم للفئة L_2

نجد كل مورفيزم $u: A \rightarrow B$ للفئة L_1 الخلفه التي تبدي $A, B \in \text{ob}(L_1)$

$$\begin{array}{ccc} F(A) & \xrightarrow{f(A)} & G(A) \\ F(u) \uparrow & & \uparrow G(u) \\ F(B) & \xrightarrow{f(B)} & G(B) \end{array}$$

لكل تبدي u أي $G(u) \circ f(B) = f(A) \circ F(u)$

المعددية

لكل فئة L لاجل كل مورفيزم $w: x \rightarrow x'$ للفئة L يوجد

$$f: h_{x'} \rightarrow h_x$$

البرهان

لفرض ان $w: x \rightarrow x'$ مورفيزم للفئة L وان $x, x' \in \text{ob}(L)$ اي كما

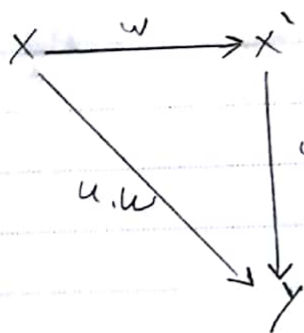
توجد دوال مباشرة $h_x, h_{x'}: L \rightarrow \text{Set}$

تعريف: $f: h_{x'} \rightarrow h_x$ التماثل $(\forall y \in \text{ob } \mathcal{L})$

$$f(y): h_{x'}(y) \rightarrow h_x(y)$$

$$f(y): \mathcal{L}(x, y) \rightarrow \mathcal{L}(x, y)$$

$$\forall u \in \mathcal{L}(x, y), f(y)(u) = u \cdot w \rightarrow \text{فاقران موزون}$$



$\mathcal{L}(x, y)$ فئة للفتحة $\mathcal{L}: Y \rightarrow Z$ و $f(y) \in \text{Mor. (sets)}$ فبا التماثل الترتيب

$$\begin{array}{ccc} h_{x'}(y) & \xrightarrow{f(y)} & h_x(y) \\ \downarrow h_{x'}(ze) & & \downarrow h_x(ze) \\ h_{x'}(z) & \xrightarrow{f(z)} & h_x(z) \end{array}$$

$$h_x(ze) \cdot f(y) = f(z) \cdot h_{x'}(ze) \quad \text{حيث } z = y$$

$$\begin{array}{ccc} \mathcal{L}(x, y) & \xrightarrow{f(y)} & \mathcal{L}(x, y) \\ \downarrow h_{x'}^*(ze) & & \downarrow h_x(ze) \\ \mathcal{L}(x, z) & \xrightarrow{f(z)} & \mathcal{L}(x, z) \end{array}$$

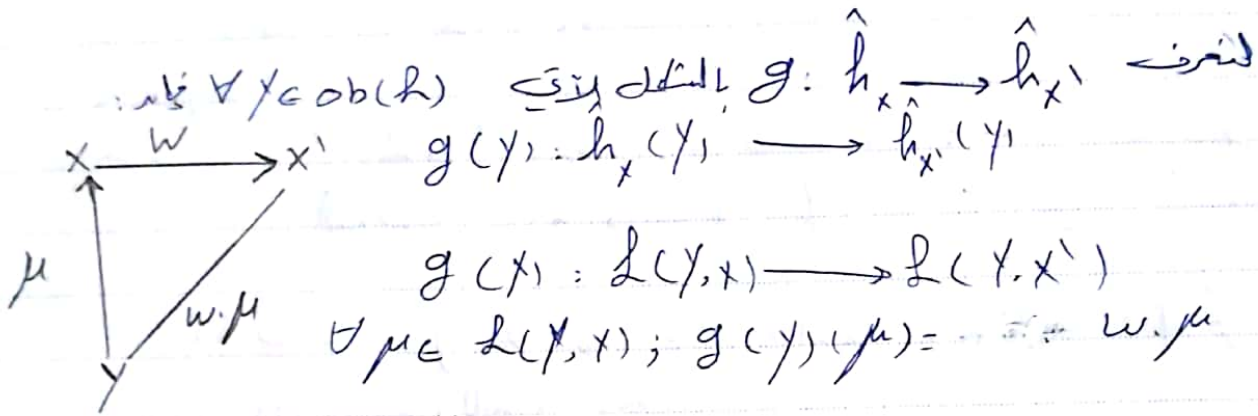
$$\begin{aligned} \forall \lambda \in \mathcal{L}(x, y); h_x(ze) \cdot f(y)(\lambda) &= h_x(ze) (f(y, \lambda)) \\ &= h_x(ze) (\lambda, ze) = ze \cdot (\lambda, w) \\ f(z) h_{x'}(ze)(\lambda) &= f(z) (h_{x'}(ze)(\lambda)) \\ &= f(z) (ze, \lambda) = (ze, \lambda) \cdot w \\ &= ze \cdot (\lambda, w) \end{aligned}$$

حيث $h_x(ze) \cdot f(y) = f(z) \cdot h_{x'}(ze)$ وحيث
بالتالي f موزون بالحق

لنفرض ان $w: X \rightarrow X'$ كالتالي
 يوجد موريزم g $\hat{h}_x \rightarrow \hat{h}_{x'}$

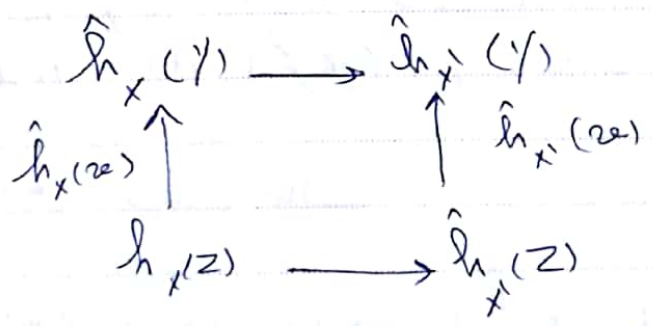
المرحان

لنفرض ان $w: X \rightarrow X'$ موريزم L فانه يمكن ان
 $x, x' \in \text{cob}(L)$ توجد $h_x, h_{x'}$ $L \rightarrow S^1$

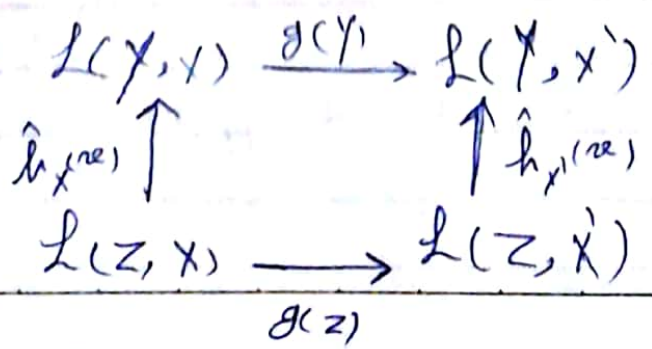


انه $g(y)$ موريزم للفترة S^1

ليكن $v: Y \rightarrow Z$ موريزم للفترة L



$\hat{h}_{x'}(ze) \cdot g(z) = g(y) \cdot \hat{h}_x(ze)$



$$\forall \lambda \in \mathcal{L}(z, x) ; \hat{h}_{x, (ze)} g(z)(\lambda) = \hat{h}_{x, (ze)} (g(z)(\lambda)) \\ = \hat{h}_{x, (ze)} (w \cdot \lambda) = (w \cdot \lambda) \cdot ze$$

$$g(y) \cdot \hat{h}_{x, (ze)}(\lambda) = g(y) (\hat{h}_{x, (ze)}(\lambda)) \\ = g(y) (\lambda \cdot ze) = (w \cdot \lambda) \cdot ze \\ = (w \cdot \lambda) \cdot ze$$

منه $\hat{h}_{x, (ze)} g(z) = g(y) \hat{h}_{x, (ze)}$
 وبالتالي g موريفزم دالي

تعريف:
 ليكن $G: \mathcal{L}_1 \rightarrow \mathcal{L}_2$ دال نقول عن الموريفزم $f: F \rightarrow G$
 دالي انه موريفزم دالي
 ايزوموريفزم

ايّا كان $A \in \text{ob}(\mathcal{L}_1)$ فان $f(A): F(A) \rightarrow G(A)$
 ايزوموريفزم للقة \mathcal{L}_2

استهبة:
 ليكن $F_1, F_2, F_3: \mathcal{L}_1 \rightarrow \mathcal{L}_2$ دال ونفرض ان $f: F_1 \rightarrow F_3$ موريفزم دالي عندئذ
 $g: F_2 \rightarrow F_3$
 $g \circ f: F_1 \rightarrow F_3$ *
 $\forall A \in \text{ob}(\mathcal{L}_1) ; (g \circ f)(A) = g(A) \cdot f(A)$
 دالي الموريفزم بالقة \mathcal{L}_2
 عندئذ $(*)$ موريفزم دالي

البرهان:
 لنفرض ان $f: F_1 \rightarrow F_2$ و $g: F_2 \rightarrow F_3$ موريفزم دالي

$$\forall A \in \text{ob}(\mathcal{L}_1) : f(A): F_1(A) \rightarrow F_2(A) \\ g(A): F_2(A) \rightarrow F_3(A)$$

موريفزم للقة \mathcal{L}_2

منه $g(A) \circ f(A): F_1(A) \rightarrow F_3(A)$
 تحققه الشرط الاول من شرط الموريفزم

ليكن $u: A \rightarrow B$ مصروفين للفضة L_1 L_2 L_3 L_4 L_5 L_6 L_7 L_8 L_9 L_{10} L_{11} L_{12} L_{13} L_{14} L_{15} L_{16} L_{17} L_{18} L_{19} L_{20} L_{21} L_{22} L_{23} L_{24} L_{25} L_{26} L_{27} L_{28} L_{29} L_{30} L_{31} L_{32} L_{33} L_{34} L_{35} L_{36} L_{37} L_{38} L_{39} L_{40} L_{41} L_{42} L_{43} L_{44} L_{45} L_{46} L_{47} L_{48} L_{49} L_{50} L_{51} L_{52} L_{53} L_{54} L_{55} L_{56} L_{57} L_{58} L_{59} L_{60} L_{61} L_{62} L_{63} L_{64} L_{65} L_{66} L_{67} L_{68} L_{69} L_{70} L_{71} L_{72} L_{73} L_{74} L_{75} L_{76} L_{77} L_{78} L_{79} L_{80} L_{81} L_{82} L_{83} L_{84} L_{85} L_{86} L_{87} L_{88} L_{89} L_{90} L_{91} L_{92} L_{93} L_{94} L_{95} L_{96} L_{97} L_{98} L_{99} L_{100} L_{101} L_{102} L_{103} L_{104} L_{105} L_{106} L_{107} L_{108} L_{109} L_{110} L_{111} L_{112} L_{113} L_{114} L_{115} L_{116} L_{117} L_{118} L_{119} L_{120} L_{121} L_{122} L_{123} L_{124} L_{125} L_{126} L_{127} L_{128} L_{129} L_{130} L_{131} L_{132} L_{133} L_{134} L_{135} L_{136} L_{137} L_{138} L_{139} L_{140} L_{141} L_{142} L_{143} L_{144} L_{145} L_{146} L_{147} L_{148} L_{149} L_{150} L_{151} L_{152} L_{153} L_{154} L_{155} L_{156} L_{157} L_{158} L_{159} L_{160} L_{161} L_{162} L_{163} L_{164} L_{165} L_{166} L_{167} L_{168} L_{169} L_{170} L_{171} L_{172} L_{173} L_{174} L_{175} L_{176} L_{177} L_{178} L_{179} L_{180} L_{181} L_{182} L_{183} L_{184} L_{185} L_{186} L_{187} L_{188} L_{189} L_{190} L_{191} L_{192} L_{193} L_{194} L_{195} L_{196} L_{197} L_{198} L_{199} L_{200} L_{201} L_{202} L_{203} L_{204} L_{205} L_{206} L_{207} L_{208} L_{209} L_{210} L_{211} L_{212} L_{213} L_{214} L_{215} L_{216} L_{217} L_{218} L_{219} L_{220} L_{221} L_{222} L_{223} L_{224} L_{225} L_{226} L_{227} L_{228} L_{229} L_{230} L_{231} L_{232} L_{233} L_{234} L_{235} L_{236} L_{237} L_{238} L_{239} L_{240} L_{241} L_{242} L_{243} L_{244} L_{245} L_{246} L_{247} L_{248} L_{249} L_{250} L_{251} L_{252} L_{253} L_{254} L_{255} L_{256} L_{257} L_{258} L_{259} L_{260} L_{261} L_{262} L_{263} L_{264} L_{265} L_{266} L_{267} L_{268} L_{269} L_{270} L_{271} L_{272} L_{273} L_{274} L_{275} L_{276} L_{277} L_{278} L_{279} L_{280} L_{281} L_{282} L_{283} L_{284} L_{285} L_{286} L_{287} L_{288} L_{289} L_{290} L_{291} L_{292} L_{293} L_{294} L_{295} L_{296} L_{297} L_{298} L_{299} L_{300} L_{301} L_{302} L_{303} L_{304} L_{305} L_{306} L_{307} L_{308} L_{309} L_{310} L_{311} L_{312} L_{313} L_{314} L_{315} L_{316} L_{317} L_{318} L_{319} L_{320} L_{321} L_{322} L_{323} L_{324} L_{325} L_{326} L_{327} L_{328} L_{329} L_{330} L_{331} L_{332} L_{333} L_{334} L_{335} L_{336} L_{337} L_{338} L_{339} L_{340} L_{341} L_{342} L_{343} L_{344} L_{345} L_{346} L_{347} L_{348} L_{349} L_{350} L_{351} L_{352} L_{353} L_{354} L_{355} L_{356} L_{357} L_{358} L_{359} L_{360} L_{361} L_{362} L_{363} L_{364} L_{365} L_{366} L_{367} L_{368} L_{369} L_{370} L_{371} L_{372} L_{373} L_{374} L_{375} L_{376} L_{377} L_{378} L_{379} L_{380} L_{381} L_{382} L_{383} L_{384} L_{385} L_{386} L_{387} L_{388} L_{389} L_{390} L_{391} L_{392} L_{393} L_{394} L_{395} L_{396} L_{397} L_{398} L_{399} L_{400} L_{401} L_{402} L_{403} L_{404} L_{405} L_{406} L_{407} L_{408} L_{409} L_{410} L_{411} L_{412} L_{413} L_{414} L_{415} L_{416} L_{417} L_{418} L_{419} L_{420} L_{421} L_{422} L_{423} L_{424} L_{425} L_{426} L_{427} L_{428} L_{429} L_{430} L_{431} L_{432} L_{433} L_{434} L_{435} L_{436} L_{437} L_{438} L_{439} L_{440} L_{441} L_{442} L_{443} L_{444} L_{445} L_{446} L_{447} L_{448} L_{449} L_{450} L_{451} L_{452} L_{453} L_{454} L_{455} L_{456} L_{457} L_{458} L_{459} L_{460} L_{461} L_{462} L_{463} L_{464} L_{465} L_{466} L_{467} L_{468} L_{469} L_{470} L_{471} L_{472} L_{473} L_{474} L_{475} L_{476} L_{477} L_{478} L_{479} L_{480} L_{481} L_{482} L_{483} L_{484} L_{485} L_{486} L_{487} L_{488} L_{489} L_{490} L_{491} L_{492} L_{493} L_{494} L_{495} L_{496} L_{497} L_{498} L_{499} L_{500} L_{501} L_{502} L_{503} L_{504} L_{505} L_{506} L_{507} L_{508} L_{509} L_{510} L_{511} L_{512} L_{513} L_{514} L_{515} L_{516} L_{517} L_{518} L_{519} L_{520} L_{521} L_{522} L_{523} L_{524} L_{525} L_{526} L_{527} L_{528} L_{529} L_{530} L_{531} L_{532} L_{533} L_{534} L_{535} L_{536} L_{537} L_{538} L_{539} L_{540} L_{541} L_{542} L_{543} L_{544} L_{545} L_{546} L_{547} L_{548} L_{549} L_{550} L_{551} L_{552} L_{553} L_{554} L_{555} L_{556} L_{557} L_{558} L_{559} L_{560} L_{561} L_{562} L_{563} L_{564} L_{565} L_{566} L_{567} L_{568} L_{569} L_{570} L_{571} L_{572} L_{573} L_{574} L_{575} L_{576} L_{577} L_{578} L_{579} L_{580} L_{581} L_{582} L_{583} L_{584} L_{585} L_{586} L_{587} L_{588} L_{589} L_{590} L_{591} L_{592} L_{593} L_{594} L_{595} L_{596} L_{597} L_{598} L_{599} L_{600} L_{601} L_{602} L_{603} L_{604} L_{605} L_{606} L_{607} L_{608} L_{609} L_{610} L_{611} L_{612} L_{613} L_{614} L_{615} L_{616} L_{617} L_{618} L_{619} L_{620} L_{621} L_{622} L_{623} L_{624} L_{625} L_{626} L_{627} L_{628} L_{629} L_{630} L_{631} L_{632} L_{633} L_{634} L_{635} L_{636} L_{637} L_{638} L_{639} L_{640} L_{641} L_{642} L_{643} L_{644} L_{645} L_{646} L_{647} L_{648} L_{649} L_{650} L_{651} L_{652} L_{653} L_{654} L_{655} L_{656} L_{657} L_{658} L_{659} L_{660} L_{661} L_{662} L_{663} L_{664} L_{665} L_{666} L_{667} L_{668} L_{669} L_{670} L_{671} L_{672} L_{673} L_{674} L_{675} L_{676} L_{677} L_{678} L_{679} L_{680} L_{681} L_{682} L_{683} L_{684} L_{685} L_{686} L_{687} L_{688} L_{689} L_{690} L_{691} L_{692} L_{693} L_{694} L_{695} L_{696} L_{697} L_{698} L_{699} L_{700} L_{701} L_{702} L_{703} L_{704} L_{705} L_{706} L_{707} L_{708} L_{709} L_{710} L_{711} L_{712} L_{713} L_{714} L_{715} L_{716} L_{717} L_{718} L_{719} L_{720} L_{721} L_{722} L_{723} L_{724} L_{725} L_{726} L_{727} L_{728} L_{729} L_{730} L_{731} L_{732} L_{733} L_{734} L_{735} L_{736} L_{737} L_{738} L_{739} L_{740} L_{741} L_{742} L_{743} L_{744} L_{745} L_{746} L_{747} L_{748} L_{749} L_{750} L_{751} L_{752} L_{753} L_{754} L_{755} L_{756} L_{757} L_{758} L_{759} L_{760} L_{761} L_{762} L_{763} L_{764} L_{765} L_{766} L_{767} L_{768} L_{769} L_{770} L_{771} L_{772} L_{773} L_{774} L_{775} L_{776} L_{777} L_{778} L_{779} L_{780} L_{781} L_{782} L_{783} L_{784} L_{785} L_{786} L_{787} L_{788} L_{789} L_{790} L_{791} L_{792} L_{793} L_{794} L_{795} L_{796} L_{797} L_{798} L_{799} L_{800} L_{801} L_{802} L_{803} L_{804} L_{805} L_{806} L_{807} L_{808} L_{809} L_{810} L_{811} L_{812} L_{813} L_{814} L_{815} L_{816} L_{817} L_{818} L_{819} L_{820} L_{821} L_{822} L_{823} L_{824} L_{825} L_{826} L_{827} L_{828} L_{829} L_{830} L_{831} L_{832} L_{833} L_{834} L_{835} L_{836} L_{837} L_{838} L_{839} L_{840} L_{841} L_{842} L_{843} L_{844} L_{845} L_{846} L_{847} L_{848} L_{849} L_{850} L_{851} L_{852} L_{853} L_{854} L_{855} L_{856} L_{857} L_{858} L_{859} L_{860} L_{861} L_{862} L_{863} L_{864} L_{865} L_{866} L_{867} L_{868} L_{869} L_{870} L_{871} L_{872} L_{873} L_{874} L_{875} L_{876} L_{877} L_{878} L_{879} L_{880} L_{881} L_{882} L_{883} L_{884} L_{885} L_{886} L_{887} L_{888} L_{889} L_{890} L_{891} L_{892} L_{893} L_{894} L_{895} L_{896} L_{897} L_{898} L_{899} L_{900} L_{901} L_{902} L_{903} L_{904} L_{905} L_{906} L_{907} L_{908} L_{909} L_{910} L_{911} L_{912} L_{913} L_{914} L_{915} L_{916} L_{917} L_{918} L_{919} L_{920} L_{921} L_{922} L_{923} L_{924} L_{925} L_{926} L_{927} L_{928} L_{929} L_{930} L_{931} L_{932} L_{933} L_{934} L_{935} L_{936} L_{937} L_{938} L_{939} L_{940} L_{941} L_{942} L_{943} L_{944} L_{945} L_{946} L_{947} L_{948} L_{949} L_{950} L_{951} L_{952} L_{953} L_{954} L_{955} L_{956} L_{957} L_{958} L_{959} L_{960} L_{961} L_{962} L_{963} L_{964} L_{965} L_{966} L_{967} L_{968} L_{969} L_{970} L_{971} L_{972} L_{973} L_{974} L_{975} L_{976} L_{977} L_{978} L_{979} L_{980} L_{981} L_{982} L_{983} L_{984} L_{985} L_{986} L_{987} L_{988} L_{989} L_{990} L_{991} L_{992} L_{993} L_{994} L_{995} L_{996} L_{997} L_{998} L_{999} L_{1000}

$$\begin{array}{ccc} F_1(A) & \xrightarrow{f(A)} & F_2(A) \\ F_1(u) \downarrow & & \downarrow F_2(u) \\ F_1(B) & \xrightarrow{f(B)} & F_2(B) \end{array}$$

$$\begin{array}{ccc} F_2(A) & \xrightarrow{g(A)} & F_3(A) \\ F_2(u) \downarrow & & \downarrow F_3(u) \\ F_2(B) & \xrightarrow{g(B)} & F_3(B) \end{array}$$

مضاهية $F_2(u) \cdot f(A) = f(B) \cdot F_1(u)$
 مضاهية $F_3(u) \cdot g(A) = g(B) \cdot F_2(u)$

لذلك يمكن كتابة المضاهية التالية:

$$\begin{array}{ccc} F_1(A) & \xrightarrow{g \cdot f(A)} & F_3(A) \\ f_1(u) \downarrow & & \downarrow F_3(u) \quad \text{--- (*)} \\ F_1(B) & \xrightarrow{g \cdot f(B)} & F_3(B) \end{array}$$

مضاهية $F_3(u) \cdot (g \cdot f)$

$$\begin{aligned} F_3(u) \cdot (g \cdot f)(A) &= F_3(u) \cdot (g(A) \cdot f(A)) \\ &= (F_3(u) \cdot g(A)) \cdot f(A) \\ &= (g(B) \cdot F_2(u)) \cdot f(A) \\ &= g(B) \cdot (F_2(u) \cdot f(A)) = g(B) \cdot (f(B) \cdot F_1(u)) \end{aligned}$$

$$= g(B) \cdot f(B) \cdot f_1(u) = (g \circ f)(B) \cdot f_1(u)$$

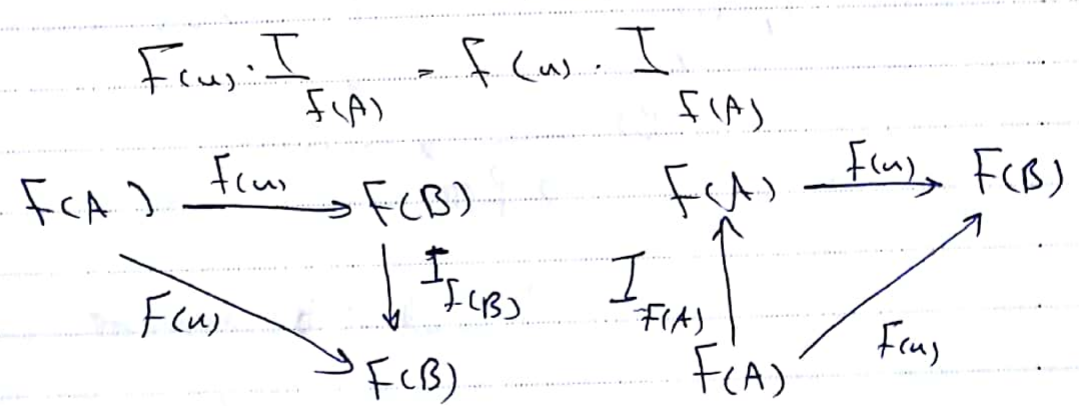
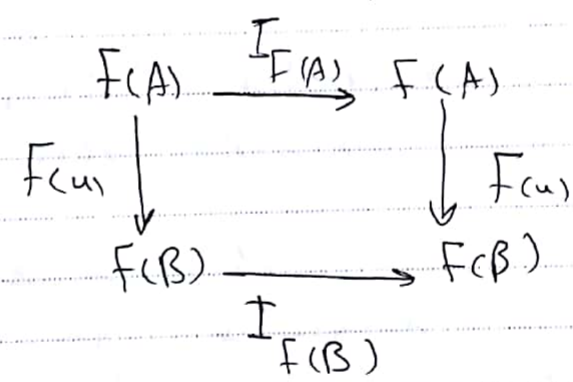
وهنا نلاحظ الخطة (تبدلي) f و g مورفزمات

لكن $f: I_1 \rightarrow I_2$ دالة خذت في الاعتبار **لتوحيد**

مورفزم دالة $I_f: F \rightarrow F$ حيث I_f مورفزم دالة مطابق

أيضا $A \in \text{ob}(I_1)$ **(البرهان)**
 $I_f(A): F(A) \rightarrow F(A) = I_{F(A)}$

الشرط المذكور محقق
 لكن $u: A \rightarrow B$ مورفزم الفئة I_1



وهنا نلاحظ الخطة تبدلي I_f مورفزم دالة F

الفئة المتماثلة